Equazioni goniometriche

Equazioni goniometriche elementari (I° tipo)

L'equazione goniometrica elementare: $\boxed{\operatorname{sen} x = m}$ ammette soluzioni se: $-1 \leq m \leq 1$

Le soluzioni sono date da:

$$\boxed{\begin{array}{l} x = \alpha + 2k180° \\ x = 180° - \alpha + 2k180° \end{array}}$$

L'equazione goniometrica elementare: $\boxed{\cos x = n}$ ammette soluzioni se: $-1 \leq n \leq 1$

Le soluzioni sono date da: $\boxed{x = \mp\alpha + 2k180°}$

L'equazione goniometrica elementare: $\boxed{\tan x = p}$ ammette soluzioni $\forall p \in R$

Le soluzioni sono date da: $\boxed{x = \alpha + k180°}$

Equazioni goniometriche elementari (II° tipo)

Le soluzioni dell''equazione: $\boxed{\operatorname{sen} x = \operatorname{sen} y}$ sono date da:

$$\boxed{\begin{array}{l} x - y = 2k180° \\ x + y = (2k+1)180° \end{array}}$$

Le soluzioni dell''equazione: $\boxed{\cos x = \cos y}$ sono date da:

$$\boxed{\begin{array}{l} x - y = 2k180° \\ x + y = 2k180° \end{array}}$$

Le soluzioni dell''equazione: $\boxed{\tan x = \tan y}$ sono date da: $\boxed{x - y = k180°}$

Esercizio 1B.146.75

$$\left|sen\left(x+60°\right)\right|=1 \qquad \left\langle \begin{matrix} sen\left(x+60°\right)=+1 \\ sen\left(x+60°\right)=-1 \end{matrix} \right. \qquad \begin{matrix} x+60°=90°+k\,360° \\ x+60°=270°+k\,360° \end{matrix} \qquad \begin{matrix} x=30°+k\,360° \\ x=210°+k\,360° \end{matrix}$$

$x=30°+k\,180°$.

Esercizio 1B.146.76

$$sen\left(x-60°\right)=sen\,2x \qquad \begin{matrix} x-60°=2x+k\,360° \\ x-60°=180°-2x+k\,360° \end{matrix} \qquad \begin{matrix} -x=60+k\,360° \\ 3x=240+k\,360° \end{matrix} \qquad \begin{matrix} x=-60°+k\,360° \\ x=80°+k\,120° \end{matrix}$$

Esercizio 1B.146.77

$$sen\left(2x-\frac{\pi}{3}\right)=sen\left(x+\frac{\pi}{6}\right) \qquad \begin{matrix} 2x-\frac{\pi}{3}=x+\frac{\pi}{6}+2k\pi \\ 2x-\frac{\pi}{3}=\pi-\left(x+\frac{\pi}{6}\right)+2k\pi \end{matrix} \qquad \begin{matrix} x=\frac{\pi}{6}+\frac{\pi}{3}+2k\pi \\ 3x=\pi-\frac{\pi}{6}+\frac{\pi}{3}+2k\pi \end{matrix}$$

$$\begin{matrix} x=\frac{\pi}{2}+2k\pi \\ 3x=\frac{7}{6}\pi+2k\pi \end{matrix} \qquad \begin{matrix} x=\frac{\pi}{2}+2k\pi \\ x=\frac{7}{18}\pi+\frac{2}{3}k\pi \end{matrix}$$

Esercizio 1B.146.80

$$cos\left(2x-15°\right)=cos\left(x+60°\right) \qquad \begin{matrix} 2x-15°=+\left(x+60°\right)+k360° \\ 2x-15°=-\left(x+60°\right)+k360° \end{matrix} \qquad \begin{matrix} x=75°+k360° \\ 3x=-45°+k360° \end{matrix} \qquad \begin{matrix} x=75°+k360° \\ x=-15°+k120° \end{matrix}$$

Equazioni lineari in seno e coseno

Un'equazione lineare in seno e coseno è un'equazione che, ridotta a forma normale, è del tipo:

$$\boxed{a\ sen\ x + b\cos x + c = 0}$$

Essa può essere trasformata nell'equazione goniometrica fondamentale:

$\boxed{A\ sen\left(x+\alpha\right)+c=0}$ *con* $\boxed{A=\sqrt{a^2+b^2}}$ *e* $\boxed{\tan\alpha = \dfrac{b}{a}}$

<u>*Dimostrazione*</u>

Applicando la formula di addizione all'equazione: $A\ sen\left(x+\alpha\right)+c=0$ *si ha:*

$A\ sen\ x\cdot\cos\alpha + A\cos x\cdot sen\alpha + c = 0$.

Quest'ultima è equivalente alla prima equazione se: $\begin{cases} A\cos\alpha = a \\ A sen\alpha = b \end{cases}$

elevando ambo i membri al quadrato si ha: $\begin{cases} A^2\cos^2\alpha = a^2 \\ A^2 sen^2\alpha = b^2 \end{cases}$

e sommando membro a membro si ottiene: $A^2\cos^2\alpha + A^2 sen^2\alpha = a^2+b^2$

cioè: $A^2\cdot\left(\cos^2\alpha + sen^2\alpha\right) = a^2+b^2$; $\quad A^2 = a^2+b^2$; $\quad A=\sqrt{a^2+b^2}$.

Mentre dal sistema: $\begin{cases} A\cos\alpha = a \\ A sen\alpha = b \end{cases}$ *dividendo membro a membro si ottiene:* $\dfrac{A sen\alpha}{A\cos\alpha} = \dfrac{b}{a}$ *cioè* $\tan\alpha = \dfrac{b}{a}$

<u>Esempio 1</u>

$$sen\ x + \sqrt{3}\cos x - 1 = 0;$$

applicando le formule: $A=\sqrt{a^2+b^2}$ *e* $\tan\alpha = \dfrac{b}{a}$

$A=\sqrt{1^2+\left(\sqrt{3}\right)^2} = 2$ *mentre* $\tan\alpha = \sqrt{3}$; $\quad \alpha = 60°$; $\quad\Rightarrow$

$2sen\left(x+60\right)=1$; $\quad sen\left(x+60\right)=\dfrac{1}{2}$

$x+60 = 30+2\,k\,180 \qquad x = -30+2\,k\,180$

$x+60 = 180-30+2\,k\,180 \qquad x = 90+2\,k\,180$

Esempio 2

$$\cos 2x - \sqrt{3}\,sen\,2x - 1 = 0;$$

$$\sqrt{3}\,sen\,2x - \cos 2x + 1 = 0$$

applicando le formule: $A = \sqrt{a^2 + b^2}$ *e* $\tan\alpha = \dfrac{b}{a}$

$A = \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} = 2$ *mentre* $\tan\alpha = -\dfrac{1}{\sqrt{3}}$; $\alpha = -30°$; $\Rightarrow$

$2sen\,(2x - 30) = -1$; $sen\,(2x - 30) = -\dfrac{1}{2}$

$2x - 30 = -30 + 2\,k\,180$	$x = k\,180$	$x = k\,180$
$2x - 30 = 180 - (-30) + 2\,k\,180$	$2x = 240 + 2\,k\,180$	$x = 120 + k\,180$

Esempio 3

$$1{,}12\,sen\,x + 0{,}97\cos x - 0{,}53 = 0;$$

applicando le formule: $A = \sqrt{a^2 + b^2}$ *e* $\tan\alpha = \dfrac{b}{a}$ *si ha:*

$A = \sqrt{1{,}12^2 + 0{,}97^2} = \sqrt{1{,}2544 + 0{,}9409} = \sqrt{2.1953} = 1{,}48$ *mentre* $\tan\alpha = \dfrac{0{,}97}{1{,}12} = 0{,}87$; $\alpha = 40{,}89°$

$\Rightarrow$ $1{,}48 \cdot sen\,(x + 40{,}89) - 0{,}53 = 0$; $sen\,(x + 40{,}89) = \dfrac{0{,}53}{1{,}48}$; $sen\,(x + 40{,}89) = 0{,}36$;

$x + 40{,}89 = 21 + 2\,k\,180$	$x = -19{,}89 + 2\,k\,180$
$x + 40{,}89 = 180 - 21 + 2\,k\,180$	$x = 118{,}11 + 2\,k\,180$

Esercizio 1B.150.181

$$sen\ x - cos\ x + 1 = 0;$$

applicando le formule: $A = \sqrt{a^2 + b^2}$ *e* $\tan\alpha = \frac{b}{a}$

$A = \sqrt{1^2 + (-1)^2} = \sqrt{2}$ *mentre* $\tan\alpha = -1$; $\alpha = -45°$; $\Rightarrow$

$\sqrt{2}sen\,(x-45) = -1$; $sen\,(x-45) = -\frac{1}{\sqrt{2}} - \frac{\sqrt{2}}{2}$

$x - 45 = -45 + 2\,k\,180$ $\quad x = 2\,k\,180$

$x - 45 = 180 - (-45) + 2\,k\,180$ $\quad x = 270 + 2\,k\,180$

Esercizio 1B.150.182

$$\sqrt{3}\cos\left(\frac{3}{2}\pi + x\right) + \cos(x - \pi) = 2;$$

Innanzitutto occorre trasformarla in una equazione lineare in seno e coseno:

$\sqrt{3}\cos(270 + x) + \cos(x - 180) = 2$; $\quad \sqrt{3}sen\ x + cos - (180 - x) = 2$; $\quad \sqrt{3}sen\ x + cos(180 - x) = 2$;

$\sqrt{3}sen\ x - cos\ x = 2$.

applicando le formule: $A = \sqrt{a^2 + b^2}$ *e* $\tan\alpha = \frac{b}{a}$

$A = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$ *mentre* $\tan\alpha = -\frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{3}$; $\alpha = -30°$; $\Rightarrow$

$2sen\,(x-30) = 2$; $sen\,(x-30) = 1$;

$x - 30 = 90 + 2\,k\,180$; $\quad x = 120 + 2\,k\,180$.

Esercizio 1B.150.184

$$cos\ x + sen\ x + 2 = 0;$$

$sen\ x + cos\ x + 2 = 0$;

applicando le formule: $A = \sqrt{a^2 + b^2}$ *e* $\tan\alpha = \frac{b}{a}$

$A = \sqrt{1^2 + 1^2} = \sqrt{2}$ *mentre* $\tan\alpha = 1$; $\alpha = 45°$; $\Rightarrow$

$\sqrt{2}sen\,(x+45) = -2$; $sen\,(x+45) = -\frac{2}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$;

Equazione impossibile poiché: $-1 \le sen\ x \le +1$ *mentre* $-\sqrt{2} < -1$.

Esercizio 1B.150.187

$$\cos\left(\frac{\pi}{3}-3x\right)-\sqrt{3}sen\left(\frac{\pi}{3}-3x\right)=1;$$

$\sqrt{3}sen(60-3x)-\cos(60-3x)=-1;$

applicando le formule: $A=\sqrt{a^2+b^2}$ *e* $\tan\alpha=\frac{b}{a}$

$A=\sqrt{\left(\sqrt{3}\right)^2+1^2}=2$ *mentre* $\tan\alpha=-\frac{1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$; $\alpha=-30°$; $\Rightarrow$

$2\cdot sen(60-3x-30)=-1$; $sen(30-3x)=-\frac{1}{2}$

$30-3x=-30+2k180$ $\quad -3x=-60+2k180$ $\quad x=20+k120$

$30-3x=180-(-30)+2k180$ $\quad -3x=180+2k180$ $\quad x=-60+k120$

Esercizio 1B.150.188

$$\sqrt{3}\cos\left(\frac{\pi}{4}-4x\right)+sen\left(\frac{\pi}{4}-4x\right)=1;$$

$sen(45-4x)+\sqrt{3}\cos(45-4x)=1$;

applicando le formule: $A=\sqrt{a^2+b^2}$ *e* $\tan\alpha=\frac{b}{a}$

$A=\sqrt{1^2+\left(\sqrt{3}\right)^2}=2$ *mentre* $\tan\alpha=\sqrt{3}$; $\alpha=60°$; $\Rightarrow$

$2sen(45-4x+60)=1$; $sen(105-4x)=\frac{1}{2}$;

$105-4x=30+2k180$ $\quad -4x=-75+2k180$ $\quad x=\frac{75}{4}+k90$

$105-4x=180-30+2k180$ $\quad -4x=45+2k180$ $\quad x=-\frac{45}{4}+k90$

Esercizio 1B.150.189

$$\cos\left(\frac{5}{6}\pi-x\right)+sen\,x-\sqrt{3}\cos x=0;$$

Innanzitutto occorre trasformarla in una equazione lineare in seno e coseno:

$\cos(150-x)+sen\,x-\sqrt{3}\cos x=0$;

$\cos150\cdot\cos x+sen150\cdot sen\,x+senx-\sqrt{3}\cos x=0$;

$-\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\cdot sen\,x+senx-\sqrt{3}\cos x=0$; $\quad \frac{3}{2}sen\,x-\frac{3\sqrt{3}}{2}\cos x=0$;

$sen\,x-\sqrt{3}\cos x=0$; *e quindi applicare le formule:* $A=\sqrt{a^2+b^2}$ *e* $\tan\alpha=\frac{b}{a}$

$A=\sqrt{1^2+\left(\sqrt{3}\right)^2}=2$ *mentre* $\tan\alpha=-\sqrt{3}$; $\alpha=-60°$; $\Rightarrow$

$2\cdot sen\left(x-60\right)=0$; $sen\left(x-60\right)=0$; $x-60=k\,180$; $x=60+k\,180$.

<u>Esercizio 1B.150.190</u>

$$\cos\left(\frac{2}{3}\pi+x\right)+4\cos x-1=0;$$

<u>Metodo 1</u>

Innanzitutto occorre trasformarla in una equazione lineare in seno e coseno:

$\cos\left(120+x\right)+4\cos x-1=0$; $\cos 120\cdot\cos x-sen\,120\cdot sen\,x+4\cos x-1=0$;

$-\frac{1}{2}\cdot\cos x-\frac{\sqrt{3}}{2}\cdot sen\,x+4\cos x-1=0$; $-\cos x-\sqrt{3}sen\,x+8\cos x-2=0$;

$\sqrt{3}sen\,x-7\cos x+2=0$; *e quindi applicare le formule parametriche razionali:*

$$\sqrt{3}\cdot\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}-7\cdot\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}+2=0\ ;\qquad 2\sqrt{3}\tan\frac{x}{2}-7+7\tan^2\frac{x}{2}+2+2\tan^2\frac{x}{2}=0\ ;$$

$$9\tan^2\frac{x}{2}+2\sqrt{3}\tan\frac{x}{2}-5=0\ ;$$

$$\tan\frac{x}{2}=\frac{-\sqrt{3}\mp\sqrt{3+45}}{9}=\frac{-\sqrt{3}\mp\sqrt{48}}{9}=\frac{-\sqrt{3}\mp4\sqrt{3}}{9}=\left\langle\begin{array}{l}=\dfrac{-\sqrt{3}-4\sqrt{3}}{9}=-\dfrac{5}{9}\sqrt{3}\\[2ex]=\dfrac{-\sqrt{3}+4\sqrt{3}}{9}=\dfrac{\sqrt{3}}{3}\end{array}\right.$$

$$\left\langle\begin{array}{l}\dfrac{x}{2}=\arctan\left(-\dfrac{5}{9}\sqrt{3}\right)+k\,180\\[2ex]\dfrac{x}{2}=30+k\,180\end{array}\right.\qquad\left\langle\begin{array}{l}x=2\arctan\left(-\dfrac{5}{9}\sqrt{3}\right)+2\,k\,180\\[2ex]x=60+2\,k\,180\end{array}\right.$$

<u>Metodo 2</u>

Innanzitutto occorre trasformarla in una equazione lineare in seno e coseno:

$\cos\left(120+x\right)+4\cos x-1=0$; $\cos 120\cdot\cos x-sen\,120\cdot sen\,x+4\cos x-1=0$;

$-\frac{1}{2}\cdot\cos x-\frac{\sqrt{3}}{2}\cdot sen\,x+4\cos x-1=0$; $-\cos x-\sqrt{3}sen\,x+8\cos x-2=0$;

$\sqrt{3}sen\,x-7\cos x+2=0$; *e quindi applicare le formule:* $A=\sqrt{a^2+b^2}$ *e* $\tan\alpha=\frac{b}{a}$

$A=\sqrt{\left(\sqrt{3}\right)^2+\left(-7\right)^2}=\sqrt{52}=2\sqrt{13}$; *mentre* $\tan\alpha=-\frac{7}{\sqrt{3}}$; $\alpha=\arctan\left(-\frac{7}{\sqrt{3}}\right)=-76{,}10°$; $\Rightarrow$

$2\sqrt{13}\cdot sen\left(x-76{,}10\right)+2=0$; $sen\left(x-76{,}10\right)=-\frac{1}{\sqrt{13}}$; $sen\left(x-76{,}10\right)=-0{,}277$;

$x-76{,}10=-16{,}10+2\,k\,180$ $\qquad x=60+2\,k\,180$

$x-76{,}10=180-\left(-16{,}10\right)+2\,k\,180$ $\qquad x=272{,}20+2\,k\,180$

Esercizio 1B.150.191

$$\sqrt{3}\cos\left(\frac{7}{6}\pi - x\right) + sen\left(\frac{11}{6}\pi - x\right) + 2 = 0;$$

Innanzitutto occorre trasformarla in una equazione lineare in seno e coseno:

$\sqrt{3}\cos(210 - x) + sen(330 - x) + 2 = 0$;

$\sqrt{3}\cdot(\cos 210\cdot\cos x + sen210\cdot sen\,x) + sen330\cdot\cos x - \cos 330\cdot senx + 2 = 0$;

$\sqrt{3}\cdot\left(-\frac{\sqrt{3}}{2}\cdot\cos x - \frac{1}{2}\cdot sen\,x\right) - \frac{1}{2}\cdot\cos x - \frac{\sqrt{3}}{2}\cdot senx + 2 = 0$;

$-\frac{3}{2}\cdot\cos x - \frac{\sqrt{3}}{2}\cdot sen\,x - \frac{1}{2}\cdot\cos x - \frac{\sqrt{3}}{2}\cdot senx + 2 = 0$;

$\sqrt{3}sen\,x + 2\cos x - 2 = 0$; *e quindi applicare le formule:* $A = \sqrt{a^2 + b^2}$ *e* $\tan\alpha = \frac{b}{a}$

$A = \sqrt{\left(\sqrt{3}\right)^2 + 2^2} = \sqrt{7}$ *mentre* $\tan\alpha = \frac{2}{\sqrt{3}}$; $\alpha = \arctan\left(\frac{2}{\sqrt{3}}\right) = 49{,}11°$; $\Rightarrow$

$\sqrt{7}\cdot sen(x + 49{,}11) - 2 = 0$; $sen(x + 49{,}11) = \frac{2}{\sqrt{7}}$; $sen(x + 49{,}11) = 0{,}76$;

$x + 49{,}11 = 49{,}11 + 2\,k\,180$ $\qquad x = 2\,k\,180$

$x + 49{,}11 = 180 - 49{,}11 + 2\,k\,180$ $\qquad x = 81{,}78 + 2\,k\,180$

Equazioni goniometriche omogenee

Un'equazione omogenea in seno e coseno è un'equazione che, ridotta a forma normale, è del tipo:

$$a\,sen^2\,x + b\,sen\,x \cdot cos\,x + c\,cos^2\,x = 0$$

Essa si trasforma in un'equazione lineare mediante le formule:

$$sen^2 x = \frac{1 - cos\,2x}{2} \qquad sen\,x \cdot cos\,x = \frac{1}{2} sen\,2x \qquad cos^2\,x = \frac{1 + cos\,2x}{2}$$

Esercizio 1B.151.198

$$sen\ x - \sqrt{3}\cos x = 0;$$

Poiché i valori $x = \frac{\pi}{2} + k\pi$ *non sono soluzioni, si divide per* $\cos x \neq 0$ *e si ottiene:*

$\tan x - \sqrt{3} = 0\ ;\quad \tan x = \sqrt{3}\ ;\quad x = 60 + k\,180\ .$

Esercizio 1B.151.199

$$sen\ (x+\alpha) - \cos(x+\alpha) = 0;$$

Poiché i valori $x = \frac{\pi}{2} + k\pi$ *non sono soluzioni, si divide per* $\cos x \neq 0$ *e si ottiene:*

$\tan(x+\alpha) - 1 = 0\ ;\quad \tan(x+\alpha) = 1\ ;\quad x + \alpha = 45 + k\,180\ ;\quad x = -\alpha + 45 + k\,180\ .$

Esercizio 1B.151.200

$$sen^2 x - 3\cos^2 x = 0;$$

Poiché i valori $x = \frac{\pi}{2} + k\pi$ *non sono soluzioni, si divide per* $\cos^2 x \neq 0$ *e si ottiene:*

$$\tan^2 x - 3 = 0\ ;\quad \left\langle \begin{array}{l} \tan x = -\sqrt{3} \\ \tan x = +\sqrt{3} \end{array}\right. ;\quad \begin{array}{l} x = -60 + k\,180 \\ x = +60 + k\,180 \end{array} .$$

Esercizio 1B.151.201

$$sen^2 x + \left(\sqrt{3} - 1\right) sen\ x \cdot \cos x - \sqrt{3}\cos^2 x = 0;$$

Poiché i valori $x = \frac{\pi}{2} + k\pi$ *non sono soluzioni, si divide per* $\cos^2 x \neq 0$ *e si ottiene:*

$\tan^2 x + \left(\sqrt{3} - 1\right)\tan x - \sqrt{3} = 0\ ;$

$$\tan x = \frac{-\sqrt{3} + 1 \mp \sqrt{\left(\sqrt{3} - 1\right)^2 + 4\sqrt{3}}}{2} = \frac{-\sqrt{3} + 1 \mp \sqrt{3 + 1 - 2\sqrt{3} + 4\sqrt{3}}}{2} = \frac{-\sqrt{3} + 1 \mp \sqrt{3 + 1 + 2\sqrt{3}}}{2} =$$

$$= \frac{-\sqrt{3} + 1 \mp \sqrt{\left(\sqrt{3} + 1\right)^2}}{2} = \frac{-\sqrt{3} + 1 \mp \left(\sqrt{3} + 1\right)}{2} = \left\langle \begin{array}{l} = \frac{-\sqrt{3} + 1 - \sqrt{3} - 1}{2} = \frac{-2\sqrt{3}}{2} = -\sqrt{3} \\ = \frac{-\sqrt{3} + 1 + \sqrt{3} + 1}{2} = \frac{2}{2} = 1 \end{array}\right.$$

$$\left\langle \begin{array}{l} \tan x = -\sqrt{3} \\ \tan x = 1 \end{array}\right. ;\quad \begin{array}{l} x = -60 + k\,180 \\ x = 45 + k\,180 \end{array} .$$

Esercizio 1B.151.202

$$2sen\,x\cdot\cos x+sen^2x=0;$$

Poiché i valori $x=\frac{\pi}{2}+k\pi$ *non sono soluzioni, si divide per* $\cos^2 x\neq 0$ *e si ottiene:*

$$2\tan x+\tan^2 x=0\,;\qquad \tan x\cdot(2+\tan x)=0\,;\qquad \begin{cases}\tan x=0\\ \tan x=-2\end{cases};\qquad \begin{array}{l}x=k\,180\\ x=\arctan(-2)+k\,180\end{array}.$$

Esercizio 1B.151.203

$$2\sqrt{2}\;sen\,x\cdot\cos x+1=0;$$

Moltiplicando 1 *per* $\cos^2 x+sen^2x=1$ *si ottiene:*

$$2\sqrt{2}\;sen\,x\cdot\cos x+1\cdot\left(\cos^2 x+sen^2x\right)=0$$

Poiché i valori $x=\frac{\pi}{2}+k\pi$ *non sono soluzioni, si divide per* $\cos^2 x\neq 0$ *e si ottiene:*

$$2\sqrt{2}\;\tan x+\tan^2 x+1=0\,;\qquad \tan^2 x+2\sqrt{2}\;\tan x+1=0\,;$$

$$\tan x=-\sqrt{2}\mp\sqrt{\left(\sqrt{2}\right)^2-1}\;=\;-\sqrt{2}\mp 1\quad \begin{cases}=\;-\sqrt{2}-1\\ =\;-\sqrt{2}+1\end{cases}\quad \begin{cases}=\;-\left(\sqrt{2}+1\right)\\ =\;-\left(\sqrt{2}-1\right)\end{cases}\quad \begin{array}{l}x=-67{,}5+k\,180\\ x=-22{,}5+k\,180\end{array}.$$

Esercizio 1B.151.204

$$2\sqrt{3}\;sen^2x-3\;senx\cdot\cos x+3\cos^2 x-\sqrt{3}=0;$$

Moltiplicando $\sqrt{3}$ *per* $\cos^2 x+sen^2x=1$ *si ottiene:*

$$2\sqrt{3}\;sen^2x-3\;senx\cdot\cos x+3\cos^2 x-\sqrt{3}\cdot\left(\cos^2 x+sen^2x\right)=0;$$

$$2\sqrt{3}\;sen^2x-3\;senx\cdot\cos x+3\cos^2 x-\sqrt{3}\cos^2 x-\sqrt{3}\;sen^2x=0;$$

$$\sqrt{3}\;sen^2x-3\;senx\cdot\cos x+\left(3-\sqrt{3}\right)\cos^2 x=0;$$

Poiché i valori $x=\frac{\pi}{2}+k\pi$ *non sono soluzioni, si divide per* $\cos^2 x\neq 0$ *e si ottiene:*

$$\sqrt{3}\;\tan^2x-3\;\tan x+\left(3-\sqrt{3}\right)=0;$$

$$\tan x=\frac{3\mp\sqrt{(-3)^2-4\sqrt{3}\cdot\left(3-\sqrt{3}\right)}}{2\sqrt{3}}=\frac{3\mp\sqrt{9-12\sqrt{3}+12}}{2\sqrt{3}}=\frac{3\mp\sqrt{\left(3-2\sqrt{3}\right)^2}}{2\sqrt{3}}=\frac{3\mp\left(3-2\sqrt{3}\right)}{2\sqrt{3}}=$$

$$\begin{cases}=\dfrac{3-3+2\sqrt{3}}{2\sqrt{3}}=\dfrac{2\sqrt{3}}{2\sqrt{3}}=1 & x=45+k\,180\\[2ex] =\dfrac{3+3-2\sqrt{3}}{2\sqrt{3}}=\dfrac{6-2\sqrt{3}}{2\sqrt{3}}=\dfrac{3-\sqrt{3}}{\sqrt{3}}=\dfrac{3-\sqrt{3}}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{3\sqrt{3}-3}{3}=\sqrt{3}-1 & x=\arctan\left(\sqrt{3}-1\right)+k\,180\end{cases}$$

Esercizio 1B.151.205

$$5\,sen^2 x - \sqrt{3}\,sen\,2x - cos^2 x = 2;$$

Moltiplicando il 2 *per* $cos^2 x + sen^2 x = 1$ *si ottiene:*

$$5\,sen^2 x - \sqrt{3}\cdot 2\,sen\,x \cdot cos\,x - cos^2 x = 2\cdot\left(cos^2 x + sen^2 x\right);$$

$$5\,sen^2 x - 2\sqrt{3}\,sen\,x\cdot cos\,x - cos^2 x - 2\,cos^2 x - 2sen^2 x = 0\,;$$

$$3\,sen^2 x - 2\sqrt{3}\,sen\,x\cdot cos\,x - 3\,cos^2 x = 0\,;$$

Poiché i valori $x = \frac{\pi}{2} + k\pi$ *non sono soluzioni, si divide per* $cos^2 x \neq 0$ *e si ottiene:*

$$3\,tan^2 x - 2\sqrt{3}\,tan\,x - 3 = 0\,;$$

$$tan\,x = \frac{\sqrt{3} \mp \sqrt{\left(\sqrt{3}\right)^2 + 9}}{3} = \frac{\sqrt{3} \mp \sqrt{12}}{3} = \frac{\sqrt{3} \mp 2\sqrt{3}}{3} = \left\langle \begin{array}{ll} \frac{\sqrt{3} - 2\sqrt{3}}{3} = -\frac{\sqrt{3}}{3} & \quad x = -30 + k\,180 \\ \frac{\sqrt{3} + 2\sqrt{3}}{3} = \sqrt{3} & \quad x = 60 + k\,180 \end{array} \right.$$

Esercizio 1B.152.231

$$cos\left(3x - \frac{\pi}{3}\right) + sen\left(\frac{5}{6}\pi - 3x\right) + 1 = 0$$

$$cos\left(3x - 60°\right) + sen\left(150° - 3x\right) + 1 = 0$$

$$cos\,3x\cdot cos\,60° + sen\,3x\cdot sen\,60° + sen\,150°\cdot cos\,3x - cos\,150°\cdot sen3x + 1 = 0\,;$$

$$\frac{1}{2}cos\,3x + \frac{\sqrt{3}}{2}sen\,3x + \frac{1}{2}cos\,3x + \frac{\sqrt{3}}{2}sen3x + 1 = 0\,;$$

$$\sqrt{3}sen\,3x + cos\,3x + 1 = 0\,;$$

$$A = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = 2\,; \qquad tan\,\gamma = \frac{1}{\sqrt{3}}\,; \qquad \gamma = 30°$$

Pertanto l'equazione si trasforma nell'equazione fondamentale:

$$2sen\left(3x + 30°\right) + 1 = 0\,; \qquad sen\left(3x + 30°\right) = -\frac{1}{2}\,;$$

$$3x + 30° = -30° + 2k\,180 \qquad 3x = -60° + 2k\,180 \qquad x = -20° + \frac{2}{3}k\,180$$

$$3x + 30° = \left(180° + 30°\right) + 2k\,180 \qquad 3x = 180° + 2k\,180 \qquad x = 60° + \frac{2}{3}k\,180$$

Esercizio 1B.152.232

$$sen\left(x+\frac{2}{3}\pi\right)+cos\left(\frac{\pi}{6}-x\right)-sen\,x-1=0$$

$sen\,(x+120°)+cos\,(30°-x)-sen\,x-1=0\,;$

$sen\,x\cdot cos120°+cos\,x\cdot sen\,120°+cos\,30°\cdot cos\,x+sen\,30°\cdot sen\,x-sen\,x-1=0\,;$

$-\frac{1}{2}\cdot sen\,x+\frac{\sqrt{3}}{2}cos\,x+\frac{\sqrt{3}}{2}\cdot cos\,x+\frac{1}{2}\cdot sen\,x-sen\,x-1=0\,;$

$-sen\,x+\sqrt{3}\,cos\,x-1=0\,;\qquad sen\,x-\sqrt{3}\,cos\,x+1=0\,;$

$A=\sqrt{1^2+\left(-\sqrt{3}\right)^2}\;=\;2\,;\qquad tan\,\gamma=-\sqrt{3}\,;\qquad \gamma=-60°$

Pertanto l'equazione si trasforma nell'equazione fondamentale:

$2sen\,(x-60°)+1=0\,;\qquad sen\,(x-60°)=-\frac{1}{2}\,;$

$x-60°=-30°+2k\,180\qquad\qquad x=30°+2k\,180$

$x-60°=(180°+30°)+2k\,180\qquad\qquad x=270+2k\,180$

Esercizio 1B.152.234

$$sen\ x \cdot (tan\ x - 1) = \sqrt{3} \cdot (sen\ x - cos\ x)$$

$sen\ x \cdot \left(\dfrac{sen\ x}{cos\ x} - 1\right) = \sqrt{3} sen\ x - \sqrt{3}\cos x\ ;\qquad \dfrac{sen^2\ x}{cos\ x} - sen\ x - \sqrt{3} sen\ x + \sqrt{3}\cos x = 0\ ;$

$sen^2 x - (1+\sqrt{3}) sen\ x \cdot cos\ x + \sqrt{3}\cos^2 x = 0\ ;$ *Applicando le formule di bisezione si ottiene:*

$\dfrac{1-cos\,2x}{2} - (1+\sqrt{3})\dfrac{1}{2} sen\ 2x + \sqrt{3}\dfrac{1+cos\,2x}{2} = 0\ ;\qquad 1 - cos\,2x - (1+\sqrt{3}) sen\ 2x + \sqrt{3}\cdot(1+cos\,2x) = 0\ ;$

$1 - cos\,2x - (1+\sqrt{3}) sen\ 2x + \sqrt{3} + \sqrt{3}\cos 2x = 0\ ;\qquad -(1+\sqrt{3}) sen\ 2x + (\sqrt{3}-1)cos\,2x + 1 + \sqrt{3} = 0\ ;$

$(1+\sqrt{3}) sen\ 2x + (1-\sqrt{3})cos\,2x - 1 - \sqrt{3} = 0\ ;$

$$A = \sqrt{(1+\sqrt{3})^2 + (1-\sqrt{3})^2} = \sqrt{1+3+2\sqrt{3}+1+3-2\sqrt{3}} = \sqrt{8} = 2\sqrt{2}$$

$tan\,\gamma = \dfrac{1-\sqrt{3}}{1+\sqrt{3}} = \dfrac{1-\sqrt{3}}{1+\sqrt{3}}\cdot\dfrac{1-\sqrt{3}}{1-\sqrt{3}} = \dfrac{1+3-2\sqrt{3}}{1-3} = \dfrac{4-2\sqrt{3}}{-2} = \sqrt{3}-2\ ;\qquad \gamma = -15°$

Pertanto l'equazione si trasforma nell'equazione fondamentale:

$2\sqrt{2} sen\,(2x - 15°) = 1 + \sqrt{3}\ ;\qquad sen\,(2x-15°) = \dfrac{1+\sqrt{3}}{2\sqrt{2}} = \dfrac{1+\sqrt{3}}{2\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{6}+\sqrt{2}}{4}\ ;$

$2x - 15° = 75° + 2k\,180\qquad 2x = 90° + 2k\,180\qquad x = 45° + k\,180$

$2x - 15° = (180° - 75°) + 2k\,180\qquad 2x = 120° + 2k\,180\qquad x = 60° + k\,180$

Esercizio 1B.153.242

$$3\ sen^2 2x - \sqrt{3} sen\ 2x \cdot cos\ 2x + 2\cos^2 2x - 3 = 0\ ;$$

Applicando le formule di bisezione si ottiene:

$$3 \cdot \frac{1-\cos 4x}{2} - \sqrt{3} \cdot \frac{1}{2} sen\ 4x + 2 \cdot \frac{1+\cos 4x}{2} - 3 = 0\ ;$$

$$3 - 3\cos 4x - \sqrt{3} sen\ 4x + 2 + 2\cos 4x - 6 = 0\ ; \qquad \sqrt{3}\ sen\ 4x + \cos 4x + 1 = 0\ ;$$

$$A = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = 2\ ; \qquad \tan\gamma = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}\ ; \qquad \gamma = 30°$$

Pertanto l'equazione si trasforma nell'equazione fondamentale:

$$2\ sen\left(4x + 30°\right) + 1 = 0\ ; \qquad sen\left(4x + 30°\right) = -\frac{1}{2}$$

$$4x + 30° = -30° + 2k\,180 \qquad 4x = -60° + 2k\,180 \qquad x = -15° + k\,90$$

$$4x + 30° = 180° + 30° + 2k\,180 \qquad 4x = 180° + 2k\,180 \qquad x = 45° + k\,90$$

Esercizio 1B.153.243

$$\tan\left(60 + x\right) = \tan\left(60 - x\right) + 4\ ;$$

$$\frac{\tan 60 + \tan x}{1 - \tan 60 \cdot \tan x} = \frac{\tan 60 - \tan x}{1 + \tan 60 \cdot \tan x} + 4\ ; \qquad \frac{\sqrt{3} + \tan x}{1 - \sqrt{3} \cdot \tan x} = \frac{\sqrt{3} - \tan x}{1 + \sqrt{3} \cdot \tan x} + 4\ ;$$

$$\left(\sqrt{3} + \tan x\right) \cdot \left(1 + \sqrt{3} \cdot \tan x\right) = \left(1 - \sqrt{3} \tan x\right) \cdot \left(\sqrt{3} - \tan x\right) + 4 \cdot \left(1 - 3\tan^2 x\right);$$

$$\sqrt{3} + 3\tan x + \tan x + \sqrt{3}\tan^2 x = \sqrt{3} - \tan x - 3\tan x + \sqrt{3}\tan^2 x + 4 - 12\tan^2 x\ ;$$

$$12\tan^2 x + 8\tan x - 4 = 0\ ; \qquad 3\tan^2 x + 2\tan x - 1 = 0\ ;$$

$$\tan x = \frac{-1 \mp \sqrt{1+3}}{3} = \frac{-1 \mp 2}{3} = \begin{cases} \dfrac{-1-2}{3} = -1 & \tan x = -1 & x = -45° + k\,180° \\ \dfrac{-1+2}{3} = \dfrac{1}{3} & \tan x = \dfrac{1}{3} & x = \arctan\dfrac{1}{3} + k\,180° \end{cases}$$

Esercizio 1B.153.244

$$sen\left(120 - 2x\right) + \cos 2x = \frac{1}{2}\ ;$$

$$sen\,120 \cdot \cos\ 2x - \cos 120 \cdot sen\ 2x + \cos 2x = \frac{1}{2}\ ; \qquad \frac{\sqrt{3}}{2}\cos\ 2x + \frac{1}{2} sen\ 2x + \cos 2x = \frac{1}{2}\ ;$$

$$\sqrt{3}\cos\ 2x + sen\ 2x + 2\cos 2x = 1\ ; \qquad sen\ 2x + \left(2 + \sqrt{3}\right)\cos 2x = 1\ ;$$

$$A = \sqrt{1^2 + \left(2 + \sqrt{3}^2\right)} = \sqrt{1 + 4 + 3 + 4\sqrt{3}} = \sqrt{8 + 4\sqrt{3}} = \sqrt{8 + \sqrt{48}} = \sqrt{\frac{8+4}{2}} + \sqrt{\frac{8-4}{2}} = \sqrt{6} + \sqrt{2}$$

$$\tan\gamma = 2 + \sqrt{3} \qquad \gamma = 75°$$

Pertanto l'equazione si trasforma nell'equazione fondamentale:

$\left(\sqrt{6}+\sqrt{2}\right) sen\left(2x+75°\right)=1$; $\quad sen\left(2x+75°\right)=\frac{1}{\sqrt{6}+\sqrt{2}}$; $\quad sen\left(2x+75°\right)=\frac{\sqrt{6}-\sqrt{2}}{4}$

$2x+75°=15°+2k\,180$ $\quad 2x=-60°+2k\,180$ $\quad x=-30°+k\,180$

$2x+75°=\left(180°-15°\right)+2k\,180$ $\quad 2x=90°+2k\,180$ $\quad x=45°+k\,180$

<u>Esercizio 1B.153.246</u>

$$\frac{sen\ 2x}{sen\ x}+\cos\left(x-\frac{4}{3}\pi\right)=\sqrt{3}\ sen\ x$$

$\frac{2\ sen\ x\cdot\cos x}{sen\ x}+\cos x\cdot\cos\frac{4}{3}\pi+sen\ x\cdot sen\frac{4}{3}\pi=\sqrt{3}\ sen\ x$;

$2\cos x-\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}sen\ x-\sqrt{3}\ sen\ x=0$; $\quad 4\cos x-\cos x-\sqrt{3}\ sen\ x-2\sqrt{3}\ sen\ x=0$;

$\sqrt{3}sen\ x-\cos x=0$;

$A=\sqrt{\left(\sqrt{3}\right)^2+\left(-1\right)^2}\ =\ 2$; $\quad \tan\gamma=-\frac{1}{\sqrt{3}}$; $\quad \gamma=-30°$

$2sen\left(x-30°\right)=0$; $\quad sen\left(x-30°\right)=0$; $\quad x-30°=k\,180°$; $\quad x=30°+k\,180°$.

<u>Esercizio 1B.153.247</u>

$$sen\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{3}+x\right)-sen\ 2x=\frac{3}{2}\cos^2 x$$

$sen\left(30°+x\right)\cdot\cos\left(60°+x\right)-sen\ 2x=\frac{3}{2}\cos^2 x$;

$\left[sen\ 30°\cdot\cos x+\cos 30° sen\ x\right]\cdot\left[\cos 60°\cdot\cos x-sen\ 60°\cdot sen\ x\right]-2sen\ x\cdot\cos x=\frac{3}{2}\cos^2 x$;

$\left[\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}sen\ x\right]\cdot\left[\frac{1}{2}\cdot\cos x-\frac{\sqrt{3}}{2}\cdot sen\ x\right]-2sen\ x\cdot\cos x=\frac{3}{2}\cos^2 x$;

$\frac{1}{4}\cos^2 x-\frac{3}{4}sen^2 x-2sen\ x\cdot\cos x=\frac{3}{2}\cos^2 x$; $\quad \cos^2 x-3sen^2 x-8sen\ x\cdot\cos x-6\cos^2 x=0$;

$3sen^2 x+8sen\ x\cdot\cos x+5\cos^2 x=0$;

Applicando le formule di bisezione si ottiene:

$3\cdot\frac{1-\cos 2x}{2}+8\cdot\frac{1}{2}sen\ 2x+5\cdot\frac{1+\cos 2x}{2}=0$; $\quad 3-3\cos 2x+8\ sen\ 2x+5+5\cos 2x=0$;

$8\ sen\ 2x+2\cos 2x+8=0$; $\quad 4\ sen\ 2x+\cos 2x+4=0$;

Trasformando l'equazione in un'equazione fondamentale si ottiene:

$A=\sqrt{4^2+1^2}\ =\ \sqrt{17}$; $\quad \tan\gamma=\frac{1}{4}$; $\quad \gamma=14{,}04°$

$\sqrt{17}\,sen\left(2x+14{,}04°\right)+4=0\ ;$ $\qquad$ $sen\left(2x+14{,}04°\right)=-\dfrac{4}{\sqrt{17}}\ ;$

$2x+14{,}04°=-75{,}96°+2\,k\,180°$ $\qquad$ $2x=-90°+2\,k\,180°$ $\qquad$ $x=-45°+k\,180°$

$2x+14{,}04°=180°-\left(-75{,}96°\right)+2\,k\,180°$ $\qquad$ $2x=241{,}92°+2\,k\,180°$ $\qquad$ $x=120{,}96°+k\,180°$

<u>Esercizio 1B.153.248</u>

$$3\ sen\left(\frac{\pi}{3}-2x\right)+2sen\ x\cdot cos\ x+cos\left(\frac{5}{6}\pi+2x\right)=1$$

$3\ sen\left(60°-2x\right)+2sen\ x\cdot cos\ x+cos\left(150°+2x\right)=1\ ;$

$3\cdot\left(sen\ 60°\cdot cos\ 2x-cos\ 60°\cdot sen\ 2x\right)+2sen\ x\cdot cos\ x+cos\,150°\cdot cos\ 2x-sen\ 150°\cdot sen\ 2x=1\ ;$

$3\cdot\left(\dfrac{\sqrt{3}}{2}\cdot cos\ 2x-\dfrac{1}{2}\cdot sen\ 2x\right)+2sen\ x\cdot cos\ x-\dfrac{\sqrt{3}}{2}\cdot cos\ 2x-\dfrac{1}{2}\cdot sen\ 2x=1\ ;$

$\dfrac{3\sqrt{3}}{2}\cdot cos\ 2x-\dfrac{3}{2}\cdot sen\ 2x+2\cdot\dfrac{1}{2}sen\ 2x-\dfrac{\sqrt{3}}{2}\cdot cos\ 2x-\dfrac{1}{2}\cdot sen\ 2x-1=0\ ;$

$3\sqrt{3}\,cos\ 2x-3sen\ 2x+2sen\ 2x-\sqrt{3}\,cos\ 2x-sen\ 2x-2=0\ ;$

$2sen\ 2x-2\sqrt{3}\,cos\ 2x+2=0\ ;$ $\qquad$ $sen\ 2x-\sqrt{3}\,cos\ 2x+1=0\ ;$

Trasformando l'equazione in un'equazione fondamentale si ottiene:

$A=\sqrt{1^2+\left(-\sqrt{3}\right)^2}\ =\ 2\ ;$ $\qquad$ $tan\,\gamma=-\sqrt{3}\ ;$ $\qquad$ $\gamma=-60°$

$2\ sen\left(2x-60°\right)+1=0\ ;$ $\qquad$ $sen\left(2x-60°\right)=-\dfrac{1}{2}\ ;$

$2x+60°=-30°+2\,k\,180°$ $\qquad$ $2x=-90°+2\,k\,180°$ $\qquad$ $x=-45°+k\,180°$

$2x+60°=180°-\left(-30°\right)+2\,k\,180°$ $\qquad$ $2x=150°+2\,k\,180°$ $\qquad$ $x=75°+k\,180°$

Esercizio 1B.153.254

$$\cos^2\left(\frac{11}{6}\pi - x\right) + \frac{\sqrt{3}}{4}\,sen\,2x = \cos^2 x$$

$\cos^2(330° - x) + \frac{\sqrt{3}}{4}\,sen\,2x = \cos^2 x$; $\qquad [\cos 330° \cos x + sen\,330° sen\,x]^2 + \frac{\sqrt{3}}{4}\,sen\,2x = \cos^2 x$;

$\left[\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\,sen\,x\right]^2 + \frac{\sqrt{3}}{4}\,sen\,2x = \cos^2 x$;

$\frac{3}{4}\cos^2 x + \frac{1}{4}sen^2 x - \frac{\sqrt{3}}{2}\,sen\,x\cos x + \frac{\sqrt{3}}{4}\,sen\,2x - \cos^2 x = 0$;

$-\frac{1}{4}\cos^2 x + \frac{1}{4}sen^2 x - \frac{\sqrt{3}}{2}\,sen\,x\cos x + \frac{\sqrt{3}}{4}\cdot 2sen\,x\cos x = 0$;

$-\frac{1}{4}\cos^2 x + \frac{1}{4}sen^2 x = 0$; $\qquad sen^2 x - \cos^2 x = 0$; $\qquad sen^2 x - (1 - sen^2 x) = 0$;

$2sen^2 x - 1 = 0$; $\qquad sen^2 x = \frac{1}{2}$; $\qquad sen\,x = \mp\frac{1}{\sqrt{2}} = \mp\frac{\sqrt{2}}{2}$;

$sen\,x = -\frac{\sqrt{2}}{2}$ $\qquad x = -45° + 2\,k\,180°$ $\qquad x = -45° + 2\,k\,180°$

$\qquad x = 180° - (-45°) + 2\,k\,180°$ $\qquad x = 225° + 2\,k\,180°$

$sen\,x = \frac{\sqrt{2}}{2}$ $\qquad x = 45° + 2\,k\,180°$ $\qquad x = 45° + 2\,k\,180°$

$\qquad x = 180° - 45° + 2\,k\,180°$ $\qquad x = 135° + 2\,k\,180°$

Esercizio 1B.153.255

$$\frac{\cos 2x}{\sqrt{2}\cos\left(\frac{\pi}{4} - x\right)} + sen\left(\frac{\pi}{2} - x\right) = 1$$

$\frac{\cos 2x}{\sqrt{2}\cos(45° - x)} + sen\,(90° - x) = 1$; $\qquad \frac{\cos^2 x - sen^2 x}{\sqrt{2}\cdot(\cos 45°\cdot\cos x + sen\,45°\cdot sen\,x)} + \cos x = 1$;

$\frac{\cos^2 x - sen^2 x}{\sqrt{2}\cdot\left(\frac{\sqrt{2}}{2}\cdot\cos x + \frac{\sqrt{2}}{2}\cdot sen\,x\right)} + \cos x = 1$; $\qquad \frac{\cos^2 x - sen^2 x}{\cos x + sen\,x} + \cos x = 1$;

$\frac{(\cos x + sen\,x)\cdot(\cos x - sen\,x)}{\cos x + sen\,x} + \cos x = 1$; $\qquad \cos x - sen\,x + \cos x = 1$; $\qquad sen\,x - 2\cos x + 1 = 0$

Trasformando l'equazione in un'equazione fondamentale si ottiene:

$A = \sqrt{1^2 + (-2)^2} = \sqrt{5}$; $\qquad \tan\gamma = -2$; $\qquad \gamma = -63{,}43°$

$\sqrt{5}\,sen\,(x - 63{,}43°) + 1 = 0$; $\qquad sen\,(x - 63{,}43°) = -\frac{1}{\sqrt{5}}$;

$x - 63{,}43° = -26{,}57° + 2\,k\,180°$ $\qquad x = 36{,}86° + 2\,k\,180°$

$x - 63{,}43° = 180° - (-26{,}57°) + 2\,k\,180°$ $\qquad x = 270° + 2\,k\,180°$

Esercizio 1B.152.233

$$sen^2 x + \left(1-\sqrt{3}\right) sen\ x \cdot cos\ x - \sqrt{3}\, cos^2\ x = 0$$

Applicando le formule di bisezione si ottiene:

$\dfrac{1-cos\,2x}{2} + \left(1-\sqrt{3}\right)\cdot \dfrac{1}{2}\, sen\ 2x - \sqrt{3}\cdot \dfrac{1+cos\,2x}{2} = 0$; $\quad 1-cos\,2x + \left(1-\sqrt{3}\right) sen\ 2x - \sqrt{3}\cdot\left(1+cos\,2x\right) = 0$;

$1-cos\,2x + \left(1-\sqrt{3}\right) sen\ 2x - \sqrt{3} - \sqrt{3}\, cos\,2x = 0$; $\quad \left(1-\sqrt{3}\right) sen\ 2x - \left(1+\sqrt{3}\right) cos\,2x + 1 - \sqrt{3} = 0$;

$$A = \sqrt{\left(1-\sqrt{3}\right)^2 + \left(1+\sqrt{3}\right)^2} = \sqrt{1+3-2\sqrt{3}+1+3+2\sqrt{3}} = \sqrt{8} = 2\sqrt{2}$$

$\tan\gamma = -\dfrac{1+\sqrt{3}}{1-\sqrt{3}} = \dfrac{1+\sqrt{3}}{\sqrt{3}-1}\cdot\dfrac{\sqrt{3}+1}{\sqrt{3}+1} = \dfrac{1+3+2\sqrt{3}}{3-1} = \dfrac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$; $\qquad \gamma = 75°$

Pertanto l'equazione si trasforma nell'equazione fondamentale:

$2\sqrt{2} sen\left(2x+75°\right) = \sqrt{3}-1$; $\quad sen\left(2x+75°\right) = \dfrac{\sqrt{3}-1}{2\sqrt{2}} = \dfrac{\sqrt{3}-1}{2\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{6}-\sqrt{2}}{4}$;

$2x + 75° = 15° + 2k\,180 \qquad 2x = -60° + 2k\,180 \qquad x = -30° + k\,180$

$2x + 75° = \left(180° - 15°\right) + 2k\,180 \qquad 2x = 90° + 2k\,180 \qquad x = 45° + k\,180$

Esercizio 1B.152.233

$$sen^2 x + \left(1-\sqrt{3}\right) sen\ x \cdot cos\ x - \sqrt{3}\, cos^2\ x = 0$$

Applicando le formule di bisezione si ottiene:

$\dfrac{1-cos\,2x}{2} + \left(1-\sqrt{3}\right)\cdot \dfrac{1}{2}\, sen\ 2x - \sqrt{3}\cdot \dfrac{1+cos\,2x}{2} = 0$; $\quad 1-cos\,2x + \left(1-\sqrt{3}\right) sen\ 2x - \sqrt{3}\cdot\left(1+cos\,2x\right) = 0$;

$1-cos\,2x + \left(1-\sqrt{3}\right) sen\ 2x - \sqrt{3} - \sqrt{3}\, cos\,2x = 0$; $\quad \left(1-\sqrt{3}\right) sen\ 2x - \left(1+\sqrt{3}\right) cos\,2x + 1 - \sqrt{3} = 0$;

$$A = \sqrt{\left(1-\sqrt{3}\right)^2 + \left(1+\sqrt{3}\right)^2} = \sqrt{1+3-2\sqrt{3}+1+3+2\sqrt{3}} = \sqrt{8} = 2\sqrt{2}$$

$\tan\gamma = -\dfrac{1+\sqrt{3}}{1-\sqrt{3}} = \dfrac{1+\sqrt{3}}{\sqrt{3}-1}\cdot\dfrac{\sqrt{3}+1}{\sqrt{3}+1} = \dfrac{1+3+2\sqrt{3}}{3-1} = \dfrac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$; $\qquad \gamma = 75°$

Pertanto l'equazione si trasforma nell'equazione fondamentale:

$2\sqrt{2} sen\left(2x+75°\right) = 1-\sqrt{3}$; $\quad sen\left(2x+75°\right) = \dfrac{1-\sqrt{3}}{2\sqrt{2}} = \dfrac{1-\sqrt{3}}{2\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = -\dfrac{\sqrt{6}-\sqrt{2}}{4}$;

$2x + 75° = -15° + 2k\,180 \qquad 2x = -90° + 2k\,180 \qquad x = -45° + k\,180$

$2x + 75° = \left(180° + 15°\right) + 2k\,180 \qquad 2x = 120° + 2k\,180 \qquad x = 60° + k\,180$