Liceo Scientifico "G. Galilei" Trebisacce
Anno Scolastico 2016-2017

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Classe: *1C Liceo Scientifico*
Prova di Matematica: *Frazioni algebriche*

1. Dopo aver determinato le condizioni di esistenza, semplifica le seguenti frazioni algebriche:

$$\frac{a^3-b^3}{a^2-b^2} \qquad \frac{10a^2-5ab-15b^2}{40a^2-100ab+60b^2} \qquad \frac{x^{-2}}{x^2}+\frac{y^{-2}}{y^2}$$

$$\frac{a^4-16}{(a^4+8a^2+16)(a^3+6a^2+12a+8)} \qquad \frac{x^3-3x+2}{x^2-4}\cdot\frac{x^2-4x+4}{3x^2+3x-6}:\left(\frac{x^2-3x+2}{3x+6}\right)^2$$

$$25y\cdot\left(\frac{x-y}{x^2-y^2}\right)^2:\frac{25x-50}{x^2-2x+xy-2y}-\left(\frac{1}{x}-\frac{1}{y}\right)\cdot\frac{xy}{y^2-x^2}$$

$$\left[\frac{1}{x+2y}-\frac{1}{x^2+4y^2+4xy}\cdot\left(x-\frac{12y^2-2x^2-2xy}{x-2y}\right)\right]:\left(\frac{1}{2y-x}+\frac{6y-x}{x^2-4y^2}\right)=$$

2. Dati due numeri reali x e y diversi da zero, sapendo che $xy=4$, calcola:

$$\left(\frac{x}{y}+\frac{y}{x}\right)^2-\frac{1}{4}\left(\frac{x^3}{y}+\frac{y^3}{x}\right)$$

Soluzione

1. Dopo aver determinato le condizioni di esistenza, semplifica le seguenti frazioni algebriche:

$$\frac{a^3-b^3}{a^2-b^2}= \qquad C.E.: \quad a \neq \mp b$$

$$=\frac{(a-b)\cdot(a^2+ab+b^2)}{(a+b)(a-b)}=\frac{a^2+ab+b^2}{a+b}\ .$$

$$\frac{10a^2-5ab-15b^2}{40a^2-100ab+60b^2}= \qquad C.E.: \quad a\neq b \ \wedge \ a\neq \frac{3}{2}b$$

$$10a^2-5ab-15b^2=5\cdot(2a^2-ab-3b^2)=5\cdot(2a^2+2ab-3ab-3b^2)=$$
$$=5\cdot[2a\cdot(a+b)-3b\cdot(a+b)]=5\cdot(a+b)\cdot(2a-3b)$$

$$40a^2-100ab+60b^2=20\cdot(2a^2-5ab+3b^2)=20\cdot(2a^2-2ab-3ab+3b^2)=$$
$$=20\cdot[2a\cdot(a-b)-3b\cdot(a-b)]=20\cdot(a-b)\cdot(2a-3b)$$

$$=\frac{5\cdot(a+b)\cdot(2a-3b)}{20\cdot(a-b)\cdot(2a-3b)}=\frac{a+b}{4\cdot(a-b)}$$

$$\frac{x^{-2}}{x^2}+\frac{y^{-2}}{y^2}= \qquad C.E.: \quad x\neq 0 \ \wedge \ y\neq 0$$

$$=\frac{1}{x^4}+\frac{1}{y^4}=\frac{y^4+x^4}{x^4y^4}\ .$$

$$\frac{a^4-16}{(a^4+8a^2+16)(a^3+6a^2+12a+8)}= \qquad C.E.: \quad a\neq -2$$

$$=\frac{(a^2+4)(a+2)(a-2)}{(a^2+4)^2\cdot(a+2)^3}=\frac{a-2}{(a+2)^2\cdot(a^2+4)}\ .$$

$$\frac{x^3-3x+2}{x^2-4}\cdot\frac{x^2-4x+4}{3x^2+3x-6}:\left(\frac{x^2-3x+2}{3x+6}\right)^2$$

$$\frac{x^3-3x+2}{x^2-4}\cdot\frac{x^2-4x+4}{3x^2+3x-6}:\left(\frac{x^2-3x+2}{3x+6}\right)^2= \qquad C.E.: x\neq\mp 2 \quad \wedge \quad x\neq 1$$

Applicando la regola di Ruffini si ha:

$$x^3-3x+2=(x-1)(x^2+x-2)=$$
$$=(x-1)(x-1)(x+2)$$

	1	0	−3	+2
1		+1	+1	−2
	1	+1	−2	0

$$=\frac{(x-1)^2(x+2)}{(x+2)(x-2)}\cdot\frac{(x-2)^2}{3(x-1)(x+2)}:\left[\frac{(x-1)(x-2)}{3(x+2)}\right]^2=$$

$$=\frac{(x-1)^2(x+2)}{(x+2)(x-2)}\cdot\frac{(x-2)^2}{3(x-1)(x+2)}:\frac{(x-1)^2(x-2)^2}{9(x+2)^2}=$$

$$= \frac{(x-1)^2(x+2)}{(x+2)(x-2)} \cdot \frac{(x-2)^2}{3(x-1)(x+2)} \cdot \frac{9(x+2)^2}{(x-1)^2(x-2)^2} =$$

$$= \frac{3(x+2)}{(x-2)(x-1)} \quad .$$

$$25y \cdot \left(\frac{x-y}{x^2-y^2}\right)^2 : \frac{25x-50}{x^2-2x+xy-2y} - \left(\frac{1}{x}-\frac{1}{y}\right) \cdot \frac{xy}{y^2-x^2} =$$

$C.E.:\ x \neq \mp y \ \wedge \ x \neq 2 \ \wedge \ x \neq 0 \ \wedge \ y \neq 0$

$$= 25y \cdot \left[\frac{x-y}{(x+y)(x-y)}\right]^2 : \frac{25(x-2)}{(x+y)(x-2)} - \frac{y-x}{xy} \cdot \frac{xy}{(y+x)(y-x)} =$$

$$= 25y \cdot \left[\frac{1}{x+y}\right]^2 : \frac{25}{x+y} - \frac{1}{x+y} =$$

$$= 25y \cdot \frac{1}{(x+y)^2} \cdot \frac{x+y}{25} - \frac{1}{x+y} =$$

$$= \frac{y}{x+y} - \frac{1}{x+y} =$$

$$= \frac{y-1}{x+y} \ .$$

$$\left[\frac{1}{x+2y} - \frac{1}{x^2+4y^2+4xy} \cdot \left(x - \frac{12y^2-2x^2-2xy}{x-2y}\right)\right] : \left(\frac{1}{2y-x} + \frac{6y-x}{x^2-4y^2}\right) =$$

$C.E.\ :\ \ x \neq \mp 2y \ \ \wedge \ \ b \neq 0$

$$= \left[\frac{1}{x+2y} - \frac{1}{(x+2y)^2} \cdot \left(\frac{x(x-2y)-12y^2+2x^2+2xy}{x-2y}\right)\right] : \left(\frac{1}{2y-x} + \frac{6y-x}{(x+2y)(x-2y)}\right) =$$

$$= \left[\frac{1}{x+2y} - \frac{1}{(x+2y)^2} \cdot \left(\frac{x^2-2xy-12y^2+2x^2+2xy}{x-2y}\right)\right] : \left(-\frac{1}{x-2y} + \frac{6y-x}{(x+2y)(x-2y)}\right) =$$

$$= \left[\frac{1}{x+2y} - \frac{1}{(x+2y)^2} \cdot \frac{3x^2-12y^2}{x-2y}\right] : \left(\frac{-(x+2y)+6y-x}{(x+2y)(x-2y)}\right) =$$

$$= \left[\frac{1}{x+2y} - \frac{1}{(x+2y)^2} \cdot \frac{3(x+2y)(x-2y)}{x-2y}\right] : \left(\frac{-x-2y+6y-x}{(x+2y)(x-2y)}\right) =$$

$$= \left[\frac{1}{x+2y} - \frac{3}{x+2y}\right] : \left(\frac{-2x+4y}{(x+2y)(x-2y)}\right) =$$

$$= \left[\frac{1-3}{x+2y}\right] : \left(\frac{-2(x-2y)}{(x+2y)(x-2y)}\right) =$$

$$= \frac{-2}{x+2y} : \frac{-2}{x+2y} \quad = \quad 1$$

Dati due numeri reali x e y diversi da zero, sapendo che $xy = 4$, calcola:

$$\left(\frac{x}{y}+\frac{y}{x}\right)^2-\frac{1}{4}\left(\frac{x^3}{y}+\frac{y^3}{x}\right)$$

Soluzione

$$\left(\frac{x}{y}+\frac{y}{x}\right)^2-\frac{1}{4}\left(\frac{x^3}{y}+\frac{y^3}{x}\right)=$$

$$=\left(\frac{x^2+y^2}{xy}\right)^2-\frac{1}{4}\left(\frac{x^4+y^4}{xy}\right)=$$

Sostituendo $xy = 4$ *si ottiene:*

$$\frac{(x^2+y^2)^2}{16}-\frac{1}{4}\cdot\frac{x^4+y^4}{4}=\frac{x^4+y^4+2x^2y^2}{16}-\frac{x^4+y^4}{16}=$$

$$=\frac{x^4+y^4+2x^2y^2-x^4-y^4}{16}=\frac{2x^2y^2}{16}=\frac{(xy)^2}{8}=\frac{16}{8}=2\ .$$