Liceo "G. Galilei" Trebisacce

Anno Scolastico 2014-2015

Prova di Matematica: *Frazioni algebriche*

Alunno: _____ Classe: *2B L. Classico*

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Tempo 60 minuti

1. Semplifica le seguenti frazioni algebriche dopo avere determinato le condizioni di esistenza:

$\frac{9a^4x^5y}{12ax^7}$ $\frac{2x-2y}{4x-4y}$ $\frac{x^2+2x}{2x}$ $\frac{3a^3-2b^2+2a^2b-3ab}{3a^3+b^2-a^2b-3ab}$ $\frac{2x^3-x^2-12x-9}{2x^3+x^2-16x-15}$

$\frac{2x+2x^2}{2x-1} : \frac{x^2-x-2}{4-8x}$ $\frac{1}{6x}+\frac{y}{3x^2}-\frac{5}{2xy}$

$\frac{a+2}{a^2+a}-\frac{1}{a}-\frac{a+1}{a^2+2a+1}$ $\left[\left(\frac{1}{a^2}-\frac{1}{b^2}\right):\left(\frac{1}{a}-\frac{1}{b}\right)\right]:\frac{a+b}{ab}$

$\frac{a+2}{a^2+a}-\frac{1}{a}-\frac{a+1}{a^2+2a+1}$ $\left[\left(\frac{1}{a^2}-\frac{1}{b^2}\right):\left(\frac{1}{a}-\frac{1}{b}\right)\right]:\frac{a+b}{ab}$

Soluzione

1. Semplifica le seguenti frazioni algebriche dopo avere determinato le condizioni di esistenza:

$$\frac{9a^4x^5y}{12ax^7} = \frac{3a^3y}{4x^2} \qquad con \quad C.E.: \quad a \neq 0 \quad \wedge \quad x \neq 0$$

$$\frac{2x-2y}{4x-4y} = \frac{2(x-y)}{4(x-y)} = \frac{1}{2} \qquad con \quad C.E.: \quad x \neq y$$

$$\frac{x^2+2x}{2x} = \frac{x(x+2)}{2x} = \frac{x+2}{2} \qquad con \quad C.E.: \quad x \neq 0$$

$$\frac{3a^3-2b^2+2a^2b-3ab}{3a^3+b^2-a^2b-3ab} = \frac{a^2(3a+2b)-b(2b+3a)}{a^2(3a-b)-b(3a-b)} = \frac{(3a+2b)(a^2-b)}{(3a-b)(a^2-b)} = \frac{3a+2b}{3a-b}$$

Con $C.E.: \quad b \neq 3a \quad \wedge \quad b \neq a^2$

$$\frac{2x^3-x^2-12x-9}{2x^3+x^2-16x-15} =$$

$2x^3-x^2-12x-9 =$ $\qquad D_9 = \{\pm 1; \pm 3; \pm 9\}$

$= (x-3)(2x^2+5x+3) =$

	2	−1	−12	−9
+3		+6	+15	+9
	2	+5	+3	=

$= (x-3)(x+1)(2x+3)\,.$

	2	+5	+3
−1		−2	−3
	2	+3	=

$2x^3+x^2-16x-15 =$ $\qquad D_{15} = \{\pm 1; \pm 3; \pm 5; \pm 15\}$

$= (x-3)(2x^2+7x+5) =$

	2	+1	−16	−15
+3		+6	+21	+15
	2	+7	+5	=

$= (x-3)(x+1)(2x+5)\,.$

	2	+7	+5
−1		−2	−5
	2	+5	=

$$= \frac{(x-3)(x+1)(2x+3)}{(x-3)(x+1)(2x+5)} = \frac{2x+3}{2x+5} \qquad con \ C.E.: \quad x \neq 3 \quad \wedge \quad x \neq -1 \quad \wedge \quad x \neq -\frac{5}{2}$$

2. Semplifica le seguenti espressioni:

$$\frac{2x+2x^2}{2x-1} : \frac{x^2-x-2}{4-8x} = \frac{2x(1+x)}{2x-1} : \frac{(x+1)(x-2)}{4(1-2x)} = \frac{2x(1+x)}{2x-1} : \frac{-4(2x-1)}{(x+1)(x-2)} = -\frac{8x}{x-2} = \frac{8x}{2-x}$$

$$C.E.: \; x \neq \frac{1}{2} \quad \wedge \quad x \neq -1 \quad \wedge \quad x \neq 2$$

$$\frac{1}{6x} + \frac{y}{3x^2} - \frac{5}{2xy} = \frac{xy+2y-15x}{6x^2y} \qquad C.E.: \; x \neq 0 \quad \wedge \quad y \neq 0$$

$$\frac{a+2}{a^2+a} - \frac{1}{a} - \frac{a+1}{a^2+2a+1} = \frac{a+2}{a(a+1)} - \frac{1}{a} - \frac{a+1}{(a+1)^2} = \frac{(a+2)(a+1)-(a+1)^2-a(a+1)}{a(a+1)^2} =$$

$$= \frac{a^2+a+2a+2-a^2-1-2a-a^2-a}{a(a+1)^2} = \frac{1-a^2}{a(a+1)^2} = \frac{(1+a)(1-a)}{a(a+1)^2} = \frac{1-a}{a(a+1)}$$

$$C.E.: \; a \neq 0 \quad \wedge \quad a \neq -1$$

$$\left[\left(\frac{1}{a^2} - \frac{1}{b^2}\right) : \left(\frac{1}{a} - \frac{1}{b}\right)\right] : \frac{a+b}{ab} = \left[\frac{b^2-a^2}{a^2b^2} : \frac{b-a}{ab}\right] : \frac{a+b}{ab} = \left[\frac{(b+a)(b-a)}{a^2b^2} \cdot \frac{ab}{b-a}\right] : \frac{a+b}{ab} =$$

$$= \frac{b+a}{ab} \cdot \frac{ab}{a+b} = 1 \; .$$

$$C.E.: \; a \neq 0 \quad \wedge \quad b \neq 0 \quad \wedge \quad a \neq \pm b$$