Liceo Scientifico "G. Galilei" Trebisacce
Anno Scolastico 2010-2011

Prova di Matematica : Frazioni algebriche

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Alunno: _____ Classe: 1 B

A. Scomponi in fattori i seguenti polinomi:

1. $a^5b^3 - a^3b^5 + b^2 - a^2$
2. $x^2 + 64 - y^2 - 16x$
3. $4xy(a^4 - b^2) + 4b^2x^2 - 4a^4x^2 - a^4y^2 + b^2y^2$
4. $x^6 - y^3 - 3x^4y + 3x^2y^2$
5. $2(x + y - 2z)^2 - 2y(x + y - 2z) + 4z(x + y - 2z)$
6. $(2x - y)^2 - x^2 - 4y^2 - 4xy$
7. $2x^4 + x^3y^4 - 16x - 8y^4$
8. $x^4 + 4x^3 + 6x^2 + 5x + 2$
9. $(4x^2 - 16)(2x + 6) + (2x^3 + 54)$
10. $3y^2 - 10y - 8$
11. $3a^4 - 12 - 3(a^3 + 2a^2 + 2a + 4)$
12. $25y^4 + \frac{36}{25}x^2y^2 - 12xy^3$

B. Determina il campo di esistenza delle seguenti frazioni algebriche:

1.	$\frac{xy}{x^2y^3 \cdot (2a - 5b)}$	
2.	$\frac{x - 3y + 1}{(x^2 + 6x + 9)}$	
3.	$\frac{2b + 1}{2x^2 + 6x}$	

C. Semplifica le seguenti frazioni algebriche:

$\frac{9x^2 + 6x + 1}{3x^2 - 3} \cdot \frac{x^2 - 1}{15x^2 + 8x + 1} =$	☐ $\frac{3x+1}{5x+1}$	☐ $\frac{3x+1}{3}$	☐ $\frac{3x+1}{x^2 \cdot (5x+1)}$	☐ $\frac{3x+1}{3(5x+1)}$
$\frac{x^3 - 6x^2 + 36x}{x^2 - 49} \cdot \frac{x^2 - x - 42}{x^4 + 216x}$	☐ $\frac{1}{x-3}$	☐ $\frac{1}{x+7}$	☐ $\frac{1}{x-7}$	☐ 1
$\frac{x + 1}{x - 2} - \frac{5 - 3x}{x + 3} - \frac{3x^2 + 7}{x^2 + x - 6} =$	☐ $\frac{x^2-7x+6}{x^2-7x+5}$	☐ $\frac{x^2-7x+6}{x^2+x-6}$	☐ $\frac{x^2-7x+6}{x^2+2x-6}$	☐ $\frac{x^2-7x+6}{x-6}$
$\frac{x^2 - 5x + 6}{x^2 - 3x + 2} \cdot \frac{x^2 - 1}{2x - 6} \cdot \frac{5}{x + 1} =$	☐ $\frac{5}{3}$	☐ $\frac{5}{2}$	☐ $x - 1$	☐ $x - 2$
$\left(\frac{2}{z + y} - \frac{2}{z - y}\right) : \frac{z^2}{z^2 - zy} =$	☐ $\frac{4y}{z(y+z)}$	☐ $\frac{y}{z(y+z)}$	☐ $-\frac{4y}{z(y+z)}$	☐ $\frac{4y}{z+y}$

D. Semplifica le seguenti frazioni algebriche:

$$\left(\frac{x^2 - 3}{3x^2 - 11x - 4} - \frac{x}{x - 4} + \frac{6 - 2x}{3x + 1}\right) : \left(\frac{2}{x - 4} - \frac{3}{1 + 3x}\right)$$

$$\left[\left(\frac{x + 1}{x - 1} - \frac{2}{x + 1}\right) \cdot \frac{x^3 - 1}{x^4 - 9} + \frac{x}{x^3 + x^2 - 3x - 3}\right] \cdot \left(-\frac{x^3 + 3x^2 - 3x - 9}{x^2 - 2x - 3}\right)$$

$$\left[\left(\frac{x}{x^2 - 1} - \frac{x}{x^2 + 1}\right) : \frac{2}{x^3 - x^2 + x - 1} + \frac{x}{x + 1}\right] : \left(\frac{3x^3}{6x^2 + 6x} \cdot \frac{2x^2 + 4x + 2}{x^3 + 2x^2 + x}\right)$$

Valutazione	Esercizio	A	B	C	D	Totale
	Punti	2 x 12	4 x 3	5 + 5 + 6 + 6 + 6	12 x 3	100
	Voto	Punteggio grezzo / 10				

Soluzione

A. Scomponi in fattori i seguenti polinomi:

1. $a^5b^3 - a^3b^5 + b^2 - a^2 =$

$= a^3b^3(a^2 - b^2) - (a^2 - b^2) =$

$= (a^2 - b^2)[a^3b^3 - 1] =$

$= (a + b)(a - b)(ab - 1)(a^2b^2 + ab + 1)$.

2. $x^2 + 64 - y^2 - 16x =$

$= (x - 8)^2 - y^2 =$

$= (x - 8 + y)(x - 8 - y)$.

3. $4xy(a^4 - b^2) + 4b^2x^2 - 4a^4x^2 - a^4y^2 + b^2y^2 =$

$= 4xy(a^4 - b^2) - 4x^2(a^4 - b^2) - y^2(a^4 - b^2) =$

$= (a^4 - b^2)(4xy - 4x^2 - y^2) =$

$= -(a^4 - b^2)(-4xy + 4x^2 + y^2) =$

$= (b^2 - a^4)(2x - y)^2 =$

$= (b + a^2)(b - a^2)(2x - y)^2$.

4. $x^6 - y^3 - 3x^4y + 3x^2y^2 =$

$= (x^2 - y)(x^4 + x^2y + y^2) - 3x^2y(x^2 - y) =$

$= (x^2 - y)(x^4 + x^2y + y^2 - 3x^2y) =$

$= (x^2 - y)(x^4 + y^2 - 2x^2y) =$

$= (x^2 - y)(x^2 - y)^2 =$

$= (x^2 - y)^3$.

5. $2(x + y - 2z)^2 - 2y(x + y - 2z) + 4z(x + y - 2z) =$

$= (x + y - 2z)[2(x + y - 2z) - 2y + 4z] =$

$= (x + y - 2z)[2x + 2y - 4z - 2y + 4z] =$

$= 2x(x + y - 2z)$.

6. $(2x-y)^2-x^2-4y^2-4xy=$

$=(2x-y)^2-(x^2+4y^2+4xy)=$

$=(2x-y)^2-(x+2y)^2=$

$=[(2x-y)+(x+2y)][(2x-y)-(x+2y)]=$

$=[2x-y+x+2y][2x-y-x-2y]=$

$=(3x+y)(x-3y)$.

7. $2x^4+x^3y^4-16x-8y^4=$

$=x^3(2x+y^4)-8(2x+y^4)=$

$=(2x+y^4)(x^3-8)=$

$=(2x+y^4)(x-2)(x^2+2x+4)=$

8. $x^4+4x^3+6x^2+5x+2=$

	1	4	6	5	2
-1		-1	-3	-3	-2
	1	3	3	2	0

$=(x+1)(x^3+3x^2+3x+2)=$

	1	3	3	2
-2		-2	-2	-2
	1	1	1	0

$=(x+1)(x+2)(x^2+x+1)$

9. $(4x^2-16)(2x+6)+(2x^3+54)=$

$=4(x^2-4)\cdot 2(x+3)+2(x^3+27)=$

$=8(x^2-4)(x+3)+2(x+3)(x^2-3x+9)=$

$=2(x+3)[4(x^2-4)+(x^2-3x+9)]=$

$=2(x+3)[4x^2-16+x^2-3x+9]=$

$=2(x+3)(5x^2-3x-7)$.

10. $3y^2-10y-8=$

$=3y^2+2y-12y-8=$

$=y(3y+2)-4(3y+2)=$

$=(3y+2)(y-4)$

$p=3\cdot(-8)=-24$		$s=-10$
+1	-24	-23
-1	+24	+23
+2	-12	-10

11. $3a^4 - 12 - 3(a^3 + 2a^2 + 2a + 4) =$

$= 3(a^4 - 4) - 3[a^2(a + 2) + 2(a + 2)] =$

$= 3(a^2 + 2)(a^2 - 2) - 3[(a + 2)(a^2 + 2)] =$

$= 3(a^2 + 2)(a^2 - 2) - 3(a + 2)(a^2 + 2) =$

$= 3(a^2 + 2)[(a^2 - 2) - (a + 2)] =$

$= 3(a^2 + 2)[a^2 - 2 - a - 2] =$

$= 3(a^2 + 2)[a^2 - a - 4]$.

12. $25y^4 + \frac{36}{25}x^2y^2 - 12xy^3 =$

$= y^2\left(25y^2 + \frac{36}{25}x^2 - 12xy\right) =$

$= y^2\left(5y - \frac{6}{5}x\right)^2$.

B. Determina il campo di esistenza delle seguenti frazioni algebriche:

$$\frac{5xy}{4x^2y^3 \cdot (2a - 5b)}$$

C.E.: $x \neq 0$

$y \neq 0$

$2a - 5b \neq 0$; $2a \neq 5b$; $a \neq \frac{5}{2}b$

$$\frac{x - 3y + 1}{(x^2 + 6x + 9)} = \frac{x - 3y + 1}{(x + 3)^2}$$

C.E.: $(x + 3)^2 \neq 0$; $x + 3 \neq 0$; $x \neq -3$

$$\frac{2b + 1}{2x^2 + 6x} = \frac{2b + 1}{2x(x + 3)}$$

C.E.: $x \neq 0$

$x + 3 \neq 0$; $x \neq -3$

C. Semplifica le seguenti frazioni algebriche:

$$\frac{9x^2+6x+1}{3x^2-3}\cdot\frac{x^2-1}{15x^2+8x+1}=$$

C.E.: $x^2-1\neq 0$; $x\neq \mp 1$

$3x+1\neq 0$; $x\neq -\frac{1}{3}$

$5x+1\neq 0$; $x\neq -\frac{1}{5}$

$$=\frac{(3x+1)^2}{3(x^2-1)}\cdot\frac{x^2-1}{(3x+1)(5x+1)}=$$

$$=\frac{3x+1}{3(5x+1)}$$

$$\frac{x^3-6x^2+36x}{x^2-49}\cdot\frac{x^2-x-42}{x^4+216x}=$$

C.E.: $x+7\neq 0$; $x\neq -7$

$x-7\neq 0$; $x\neq +7$

$x\neq 0$

$x+6\neq 0$; $x\neq -6$

$$=\frac{x(x^2-6x+36)}{(x+7)(x-7)}\cdot\frac{(x+6)(x-7)}{x(x^3+216)}=$$

$$=\frac{x(x^2-6x+36)}{(x+7)(x-7)}\cdot\frac{(x+6)(x-7)}{x(x+6)(x^2-6x+36)}=$$

$$=\frac{1}{x+7}$$

$$\frac{x+1}{x-2}-\frac{5-3x}{x+3}-\frac{3x^2+7}{x^2+x-6}=$$

C.E.: $x-2\neq 0$; $x\neq +2$

$x+3\neq 0$; $x\neq -3$

$$=\frac{x+1}{x-2}-\frac{5-3x}{x+3}-\frac{3x^2+7}{(x-2)(x+3)}=$$

$$=\frac{(x+3)(x+1)-(x-2)(5-3x)-(3x^2+7)}{(x-2)(x+3)}=$$

$$=\frac{x^2+x+3x+3-(5x-3x^2-10+6x)-3x^2+7}{(x-2)(x+3)}=$$

$$=\frac{x^2+x+3x+3-5x+3x^2+10-6x-3x^2+7}{(x-2)(x+3)}=$$

$$=\frac{x^2-7x+20}{(x-2)(x+3)}=$$

$$=\frac{x^2-7x+6}{x^2+x-6}\ .$$

$$\frac{x^2-5x+6}{x^2-3x+2}\cdot\frac{x^2-1}{2x-6}\cdot\frac{5}{x+1}=$$

C.E.: $x-2\neq 0\,;\quad x\neq +2$

$x-1\neq 0\,;\quad x\neq +1$

$x+1\neq 0\,;\quad x\neq -1$

$x-3\neq 0\,;\quad x\neq +3$

$$=\frac{(x-2)(x-3)}{(x-2)(x-1)}\cdot\frac{(x+1)(x-1)}{2(x-3)}\cdot\frac{5}{x+1}=$$

$$=\frac{5}{2}\,.$$

$$\left(\frac{2}{z+y}-\frac{2}{z-y}\right):\frac{z^2}{z^2-zy}=$$

C.E.: $z\neq 0$

$z+y\neq 0\,;\quad z\neq -y$

$z-y\neq 0\,;\quad z\neq +y$

$$=\frac{2(z-y)-2(z+y)}{(z+y)(z-y)}:\frac{z^2}{z(z-y)}=$$

$$=\frac{-4y}{(z+y)(z-y)}\cdot\frac{z(z-y)}{z^2}=$$

$$=\frac{-4y}{z(z+y)}\,.$$

D. Semplifica le seguenti frazioni algebriche:

$$\left(\frac{x^2-3}{3x^2-11x-4}-\frac{x}{x-4}+\frac{6-2x}{3x+1}\right):\left(\frac{2}{x-4}-\frac{3}{1+3x}\right)=$$

C.E.: $x-4\neq 0$; $x\neq +4$

$3x+1\neq 0$; $x\neq -\frac{1}{3}$

$3x+14\neq 0$; $x\neq -\frac{14}{3}$

$$=\left(\frac{x^2-3}{(x-4)(3x+1)}-\frac{x}{x-4}+\frac{6-2x}{3x+1}\right):\left(\frac{2}{x-4}-\frac{3}{1+3x}\right)=$$

$$=\frac{x^2-3-x(3x+1)+(6-2x)(x-4)}{(x-4)(3x+1)}:\frac{2(3x+1)-3(x-4)}{(x-4)(1+3x)}=$$

$$=\frac{x^2-3-3x^2-x+6x-24-2x^2+8x}{(x-4)(3x+1)}:\frac{6x+2-3x+12}{(x-4)(1+3x)}=$$

$$=\frac{-4x^2+13x-27}{(x-4)(3x+1)}:\frac{3x+14}{(x-4)(1+3x)}=$$

$$=\frac{-4x^2+13x-27}{(x-4)(3x+1)}\cdot\frac{(x-4)(1+3x)}{3x+14}=$$

$$=\frac{-4x^2+13x-27}{3x+14}\ .$$

$$\left[\left(\frac{x+1}{x-1}-\frac{2}{x+1}\right)\cdot\frac{x^3-1}{x^4-9}+\frac{x}{x^3+x^2-3x-3}\right]\cdot\left(-\frac{x^3+3x^2-3x-9}{x^2-2x-3}\right)=$$

C.E.: $x+1\neq 0$; $x\neq -1$

$x-1\neq 0$; $x\neq +1$

$x-3\neq 0$; $x\neq +3$

$x^2-3\neq 0$; $x\neq \pm\sqrt{3}$

$$=\left[\frac{(x+1)(x+1)-2(x-1)}{(x+1)(x-1)}\cdot\frac{(x-1)(x^2+x+1)}{(x^2+3)(x^2-3)}+\frac{x}{(x+1)(x^2-3)}\right]\cdot\left[-\frac{(x+3)(x^2-3)}{(x+1)(x-3)}\right]=$$

$$=\left[\frac{x^2+1+2x-2x+2}{(x+1)(x-1)}\cdot\frac{(x-1)(x^2+x+1)}{(x^2+3)(x^2-3)}+\frac{x}{(x+1)(x^2-3)}\right]\cdot\left[-\frac{(x+3)(x^2-3)}{(x+1)(x-3)}\right]=$$

$$=\left[\frac{x^2+3}{(x+1)(x-1)}\cdot\frac{(x-1)(x^2+x+1)}{(x^2+3)(x^2-3)}+\frac{x}{(x+1)(x^2-3)}\right]\cdot\left[-\frac{(x+3)(x^2-3)}{(x+1)(x-3)}\right]=$$

$$=\left[\frac{x^2+x+1}{(x+1)(x^2-3)}+\frac{x}{(x+1)(x^2-3)}\right]\cdot\left[-\frac{(x+3)(x^2-3)}{(x+1)(x-3)}\right]=$$

$$=\left[\frac{x^2+x+1+x}{(x+1)(x^2-3)}\right]\cdot\left[-\frac{(x+3)(x^2-3)}{(x+1)(x-3)}\right]=$$

$$=-\frac{(x+1)^2}{(x+1)(x^2-3)}\cdot\frac{(x+3)(x^2-3)}{(x+1)(x-3)}=$$

$$=-\frac{x+3}{x-3}\quad=\quad\frac{x+3}{3-x}\ .$$

$$\left[\left(\frac{x}{x^2-1}-\frac{x}{x^2+1}\right):\frac{2}{x^3-x^2+x-1}+\frac{x}{x+1}\right]:\left(\frac{3x^3}{6x^2+6x}\cdot\frac{2x^2+4x+2}{x^3+2x^2+x}\right)=$$

C.E.: $x+1\neq 0\,;\quad x\neq -1$

$x-1\neq 0\,;\quad x\neq +1$

$x\neq 0\,;$

$$=\left[\frac{x(x^2+1)-x(x^2-1)}{(x^2+1)(x^2-1)}:\frac{2}{(x^2+1)(x-1)}+\frac{x}{x+1}\right]:\left(\frac{3x^3}{6x(x+1)}\cdot\frac{2(x^2+2x+1)}{x(x^2+2x+1)}\right)=$$

$$=\left[\frac{x^3+x-x^3+x}{(x^2+1)(x^2-1)}\cdot\frac{(x^2+1)(x-1)}{2}+\frac{x}{x+1}\right]:\frac{x}{(x+1)}=$$

$$=\left[\frac{2x}{(x^2+1)(x+1)(x-1)}\cdot\frac{(x^2+1)(x-1)}{2}+\frac{x}{x+1}\right]:\frac{x}{(x+1)}=$$

$$=\left[\frac{x}{x+1}+\frac{x}{x+1}\right]:\frac{x}{(x+1)}=$$

$$=\frac{2x}{x+1}\cdot\frac{x+1}{x}=$$

$$=2\ .$$