LIMITI

Calcolo di limiti di funzioni esponenziali e logaritmiche

Il calcolo di gran parte dei limiti delle funzioni esponenziali e logaritmiche si effettua utilizzando i seguenti limiti notevoli:

Limite notevole	$\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x = e$

$e = 2{,}71828182\ldots$

Da dimostrare

Limite notevole	$\lim_{x\to 0}\frac{ln(1+x)}{x} = 1$

Dimostrazione

$$\lim_{x\to0}\frac{ln\,(1+x)}{x} = \left(\frac{0}{0}=?\right)$$

Applichiamo la proprietà dei logaritmi:

$$\lim_{x\to0}\frac{ln\,(1+x)}{x} = \lim_{x\to0}\frac{1}{x}\cdot ln\,(1+x) = \lim_{x\to0} ln\,(1+x)^{\frac{1}{x}} =$$

Per la continuità della funzione logaritmica:

$$\lim_{x\to0} ln\,(1+x)^{\frac{1}{x}} = ln\left[\lim_{x\to0}\,(1+x)^{\frac{1}{x}}\right] =$$

Poniamo $\frac{1}{x}=t \quad\Rightarrow\quad x=\frac{1}{t}$. Per $x\to0 \qquad t\to\pm\infty$

Sostituendo si ha:

$$ln\left[\lim_{x\to0}\,(1+x)^{\frac{1}{x}}\right] = ln\left[\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^t\right] = ln\,e = 1\,.$$

Limite notevole	$\lim_{x\to 0}\frac{e^x-1}{x} = 1$

Dimostrazione

$$\lim_{x\to0}\frac{e^x-1}{x} = \left(\frac{0}{0}=?\right)$$

Poniamo $e^x-1=t \quad\Rightarrow\quad e^x=1+t\,; \qquad x=ln\,(1+t)$ Per $x\to0 \qquad t\to0$

Sostituendo si ha:

$$\lim_{t\to0}\frac{t}{ln\,(1+t)} = \lim_{t\to0}\frac{1}{\frac{ln\,(1+t)}{t}} = \frac{1}{1} = 1\,.$$

Esercizio 1268.265

$$\lim_{x\to +\infty} \left(\frac{2+x}{x}\right)^x = (1^\infty = ?)$$

$$\lim_{x\to +\infty} \left(\frac{2+x}{x}\right)^x = \lim_{x\to +\infty} \left(\frac{2}{x}+\frac{x}{x}\right)^x = \lim_{x\to +\infty} \left(1+\frac{2}{x}\right)^x$$

Poniamo $\frac{2}{x}=\frac{1}{t}$ $\Rightarrow$ $x=2t$ Per $x\to+\infty$ $t\to+\infty$

Sostituendo si ha:

$$\lim_{x\to +\infty} \left(1+\frac{2}{x}\right)^x = \lim_{t\to +\infty} \left(1+\frac{1}{t}\right)^{2t} = \lim_{t\to +\infty} \left[\left(1+\frac{1}{t}\right)^t\right]^2 = e^{\,2} \,.$$

Esercizio 1269.266

$$\lim_{x\to -\infty} \left(\frac{x-2}{x}\right)^x = (1^\infty = ?)$$

$$\lim_{x\to -\infty} \left(\frac{x-2}{x}\right)^x = \lim_{x\to -\infty} \left(\frac{x}{x}-\frac{2}{x}\right)^x = \lim_{x\to -\infty} \left(1-\frac{2}{x}\right)^x$$

Poniamo $-\frac{2}{x}=\frac{1}{t}$ $\Rightarrow$ $x=-2t$ Per $x\to-\infty$ $t\to+\infty$

Sostituendo si ha:

$$\lim_{x\to -\infty} \left(1-\frac{2}{x}\right)^x = \lim_{t\to +\infty} \left(1+\frac{1}{t}\right)^{-2t} = \lim_{t\to +\infty} \left[\left(1+\frac{1}{t}\right)^t\right]^{-2} = e^{-2} = \frac{1}{e^2} \,.$$

Esercizio 1269.266.b

$$\lim_{x\to -\infty} \left(\frac{x-2}{x}\right)^{3x} = (1^\infty = ?)$$

$$\lim_{x\to -\infty} \left(\frac{x-2}{x}\right)^{3x} = \lim_{x\to -\infty} \left(\frac{x}{x}-\frac{2}{x}\right)^{3x} = \lim_{x\to -\infty} \left(1-\frac{2}{x}\right)^{3x}$$

Poniamo $-\frac{2}{x}=\frac{1}{t}$ $\Rightarrow$ $x=-2t$ Per $x\to-\infty$ $t\to+\infty$

Sostituendo si ha:

$$\lim_{x\to +\infty} \left(1-\frac{2}{x}\right)^{3x} = \lim_{t\to +\infty} \left(1+\frac{1}{t}\right)^{-2t\cdot 3} = \lim_{t\to +\infty} \left[\left(1+\frac{1}{t}\right)^t\right]^{-6} = e^{-6} = \frac{1}{e^6} \,.$$

Esercizio 1269.266.c

$$\lim_{x\to -\infty} \left(\frac{x-2}{x}\right)^{3x-5} = (1^\infty = ?)$$

$$\lim_{x\to -\infty} \left(\frac{x-2}{x}\right)^{3x-5} = \lim_{x\to -\infty} \left(\frac{x}{x}-\frac{2}{x}\right)^{3x-5} = \lim_{x\to -\infty} \left(1-\frac{2}{x}\right)^{3x-5}$$

Poniamo $-\frac{2}{x}=\frac{1}{t}$ $\Rightarrow$ $x=-2t$ Per $x\to-\infty$ $t\to+\infty$ Sostituendo si ha:

$$\lim_{x\to -\infty} \left(1-\frac{2}{x}\right)^{3x-5} = \lim_{x\to -\infty} \left(1-\frac{2}{x}\right)^{3x} : \left(1-\frac{2}{x}\right)^5 = \lim_{t\to +\infty} \left(1+\frac{1}{t}\right)^{-2t\cdot 3} : \left(1+\frac{1}{t}\right)^5 =$$

$$= \lim_{t\to +\infty} \left[\left(1+\frac{1}{t}\right)^t\right]^{-6} : \left(1+\frac{1}{t}\right)^5 = e^{-6} : 1 = \frac{1}{e^6} \,.$$

Esercizio 1269.268

$$\lim_{x\to -\infty} \{x[ln(x+1) - ln\, x]\} \;=\; (\infty \cdot 0 = ?)$$

$$\lim_{x\to -\infty} \{x[ln(x+1) - ln\, x]\} \;=\; \lim_{x\to -\infty} \left(x \cdot ln\frac{1+x}{x}\right) \;=\; \lim_{x\to -\infty} ln\left(1+\frac{1}{x}\right)^x \;=\; ln\, e \;=\; 1\ .$$

Esercizio 1269.271

$$\lim_{x\to 0} \frac{ln(x+5) - ln\, 5}{x} \;=\; \left(\frac{0}{0} = ?\right)$$

$$\lim_{x\to 0} \frac{ln(x+5) - ln\, 5}{x} \;=\; \lim_{x\to 0} \frac{ln\frac{(x+5)}{5}}{x} \;=\; \lim_{x\to 0} \frac{1}{x} \cdot ln\frac{x+5}{5} \;=\; \lim_{x\to 0}\ ln\left(1+\frac{x}{5}\right)^{\frac{1}{x}} \;=$$

Poniamo $\frac{x}{5} = \frac{1}{t}$ $\Rightarrow$ $x = \frac{5}{t}$ Per $x \to 0$ $t \to \infty$ Sostituendo si ha:

$$\lim_{x\to 0}\ ln\left(1+\frac{x}{5}\right)^{\frac{1}{x}} \;=\; \lim_{t\to \infty}\ ln\left(1+\frac{1}{t}\right)^{\frac{1}{\frac{5}{t}}} \;=\; \lim_{t\to \infty}\ ln\left(1+\frac{1}{t}\right)^{\frac{t}{5}} \;=\; \lim_{t\to \infty}\ ln\left[\left(1+\frac{1}{t}\right)^t\right]^{\frac{1}{5}} \;=\; ln\, e^{\frac{1}{5}} \;=\; \frac{1}{5}\ .$$

Esercizio 1269.273

$$\lim_{x\to 0^+} \frac{e^{\sqrt{3x}} - 1}{5\sqrt{x}} \;=\; \left(\frac{0}{0} = ?\right)$$

$$\lim_{x\to 0^+} \frac{e^{\sqrt{3x}} - 1}{5\sqrt{x}} \;=\; \lim_{x\to 0^+} \frac{e^{\sqrt{3x}} - 1}{\sqrt{3x}} \cdot \frac{\sqrt{3}}{5} \;=\; 1 \cdot \frac{\sqrt{3}}{5} \;=\; \frac{\sqrt{3}}{5}\ .$$

Esercizio 1269.274

$$\lim_{x\to +\infty} \left(\frac{2x+3}{2x-1}\right)^{3x+4} \;=\; (1^\infty = ?)$$

$$\lim_{x\to +\infty} \left(\frac{2x+3}{2x-1}\right)^{3x+4} \;=\; \lim_{x\to +\infty} \left(\frac{2x-1+4}{2x-1}\right)^{3x+4} \qquad \textit{Abbiamo sostituito} \quad +3 = -1+4$$

$$=\; \lim_{x\to +\infty} \left(\frac{2x-1}{2x-1} + \frac{4}{2x-1}\right)^{3x+4} \;=\; \lim_{x\to +\infty} \left(1+\frac{4}{2x-1}\right)^{3x+4} \;=$$

Poniamo $\frac{4}{2x-1} = \frac{1}{t}$ $\Rightarrow$ $2x-1 = 4t\,;$ $2x = 4t+1\,;$ $x = 2t+\frac{1}{2}$ Per $x \to +\infty$ $t \to +\infty$

Sostituendo si ha:

$$\lim_{x\to +\infty} \left(1+\frac{4}{2x-1}\right)^{3x+4} \;=\; \lim_{t\to +\infty} \left(1+\frac{1}{t}\right)^{3\cdot\left(2t+\frac{1}{2}\right)} \;=\; \lim_{t\to +\infty} \left(1+\frac{1}{t}\right)^{6t+\frac{3}{2}} \;=\; .$$

$$=\; \lim_{t\to +\infty} \left(1+\frac{1}{t}\right)^{6t} \cdot \left(1+\frac{1}{t}\right)^{\frac{3}{2}} \;=\; \lim_{t\to +\infty} \left|\left(1+\frac{1}{t}\right)^t\right|^6 \cdot \left(1+\frac{1}{t}\right)^{\frac{3}{2}} \;=\; e^{\,6}\ .$$

Esercizio 1269.276

$$\lim_{x\to \infty} 3x \cdot \left(1 - e^{\frac{1}{5x}}\right) \;=\; (\infty \cdot 0 = ?)$$

Poniamo $\frac{1}{5x} = t$ $\Rightarrow$ $x = \frac{1}{5t}$ Per $x \to \infty$ $t \to 0$

$$\lim_{x\to \infty} 3x \cdot \left(1 - e^{\frac{1}{5x}}\right) \;=\; \lim_{t\to 0} 3 \cdot \frac{1}{5t} \cdot (1 - e^t) \;=\; \lim_{t\to 0} \frac{3}{5} \cdot \frac{(1-e^t)}{t} \;=\; \lim_{t\to 0} -\frac{3}{5} \cdot \frac{e^t - 1}{t} \;=\; -\frac{3}{5}\ .$$

Esercizio 1269.279

$$\lim_{x\to -3} \frac{e^{2x+6}-1}{x+3} \;=\; \left(\frac{0}{0}=?\right)$$

Poniamo $x+3=t \quad \Rightarrow \quad x=t-3 \qquad$ Per $x\to -3 \quad t\to 0$

$$\lim_{x\to -3} \frac{e^{2(x+3)}-1}{x+3} \;=\; \lim_{t\to 0} \frac{e^{2t}-1}{t} \;=\; \lim_{t\to 0} \frac{e^{2t}-1}{2t}\cdot 2 \;=1\cdot 2=2\ .$$

Esercizio 1269.281

$$\lim_{x\to 0} \frac{ln\ (1-5x)}{3x} \;=\; \left(\frac{0}{0}=?\right)$$

Poniamo $-5x=t \quad \Rightarrow \quad x=-\frac{t}{5} \qquad$ Per $x\to 0 \quad t\to 0 \qquad$ Sostituendo si ha:

$$\lim_{x\to 0} \frac{ln\ (1-5x)}{3x} \;=\; \lim_{t\to 0} \frac{ln\ (1+t)}{3\cdot\left(-\frac{t}{5}\right)} \;=\; \lim_{t\to 0} \frac{ln\ (1+t)}{-\frac{3}{5}\cdot t} \;=\; \lim_{t\to 0} -\frac{5}{3}\cdot\frac{ln\ (1+t)}{t} \;=\; -\frac{5}{3}\cdot 1 \;=\; -\frac{5}{3}\ .$$

Esercizio 1269.284

$$\lim_{x\to 0} \frac{4\,ln\ (1+3x)}{sin\,5x} \;=\; \left(\frac{0}{0}=?\right)$$

$$\lim_{x\to 0} \frac{4\,ln\ (1+3x)}{sin\,5x} \;=\; \lim_{x\to 0} \frac{5x}{sin\,5x}\cdot\frac{ln\ (1+3x)}{3x}\cdot\frac{3\cdot 4}{5} \;=\; 1\cdot 1\cdot\frac{12}{5} \;=\; \frac{12}{5}\ .$$

Esercizio 1269.285

$$\lim_{x\to 1} \frac{e^{x}-e}{3x-3} \;=\; \left(\frac{0}{0}=?\right)$$

Poniamo $x-1=t \quad \Rightarrow \quad x=1+t \qquad$ Per $x\to 1 \quad t\to 0$

$$\lim_{x\to 1} \frac{e^{x}-e}{3x-3} \;=\; \lim_{t\to 0} \frac{e^{1+t}-e}{3(1+t)-3} \;=\; \lim_{t\to 0} \frac{e\cdot e^{t}-e}{3+3t-3} \;=\; \lim_{t\to 0} \frac{e\cdot(e^{t}-1)}{3t} \;=\; \lim_{t\to 0} \frac{e}{3}\cdot\frac{e^{t}-1}{t} \;=\; \frac{e}{3}\cdot 1 \;=\; \frac{e}{3}$$

Esercizio 1269.286

$$\lim_{x\to +\infty} 2x\cdot ln\frac{5x+1}{5x} \;=\; (\infty\cdot 0=?)$$

$$\lim_{x\to +\infty} 2x\cdot ln\frac{5x+1}{5x} \;=\; \lim_{x\to +\infty} 2\cdot ln\left(\frac{5x+1}{5x}\right)^{x} \;=\; \lim_{x\to +\infty} 2\cdot ln\left(1+\frac{1}{5x}\right)^{x} \;=$$

$$=\; \lim_{x\to +\infty} 2\cdot\left[ln\left(1+\frac{1}{5x}\right)^{5x}\right]^{\frac{1}{5}} \;=\; 2\cdot ln\ e^{\frac{1}{5}} \;=\; 2\cdot\frac{1}{5} \;=\; \frac{2}{5}\ .$$

Esercizio 1269.288

$$\lim_{x\to 0} \frac{cos\,x-ln(1+x)-1}{5x} \;=\; \left(\frac{0}{0}=?\right)$$

$$\lim_{x\to 0} \frac{cos\,x-ln(1+x)-1}{5x} \;=\; \lim_{x\to 0} \frac{1}{5}\cdot\left[-\frac{1-cos\,x}{x}-\frac{ln(1+x)}{x}\right] \;=\; \frac{1}{5}\cdot(0-1)=-\frac{1}{5}$$

Esercizio 1269.289

$$\lim_{x\to 0} \frac{e^x - e^{-x}}{6x} = \left(\frac{0}{0} = ?\right)$$

$$\lim_{x\to 0} \frac{e^x - e^{-x}}{6x} = \lim_{x\to 0} \frac{e^x - e^{-x} - 1 + 1}{6x} = \lim_{x\to 0} \frac{1}{6} \cdot \left[\frac{e^x - 1}{x} + \frac{-e^{-x} + 1}{x}\right] = \lim_{x\to 0} \frac{1}{6} \cdot \left[\frac{e^x - 1}{x} + \frac{-(e^{-x} - 1)}{x}\right] =$$

$$= \lim_{x\to 0} \frac{1}{6} \cdot \left[\frac{e^x - 1}{x} + \frac{e^{-x} - 1}{-x}\right] = \frac{1}{6} \cdot [1 + 1] = \frac{2}{6} = \frac{1}{3} .$$

Esercizio 1269.290

$$\lim_{x\to 0} \frac{1 - cos^3 x}{e^{5x^2} - 1} = \left(\frac{0}{0} = ?\right)$$

$$\lim_{x\to 0} \frac{1 - cos^3 x}{e^{5x^2} - 1} = \lim_{x\to 0} \frac{(1 - cos\, x) \cdot (1 + cos\, x + cos^2 x)}{e^{5x^2} - 1} =$$

$$= \lim_{x\to 0} \frac{(1 - cos\, x)}{x^2} \cdot \frac{5x^2}{e^{5x^2} - 1} \cdot \frac{1}{5} \cdot (1 + cos\, x + cos^2 x) = \frac{1}{2} \cdot 1 \cdot \frac{1}{5} \cdot 3 = \frac{3}{10} .$$

Esercizio 1269.291

$$\lim_{x\to 0^+} [ln(tg\, x) - ln(5x)] = (-\infty + \infty = ?)$$

$$\lim_{x\to 0^+} [ln(tg\, x) - ln(5x)] = \lim_{x\to 0^+} ln\left(\frac{tg\, x}{5x}\right) = \lim_{x\to 0^+} ln\left(\frac{1}{5} \cdot \frac{tg\, x}{x}\right) = ln\left(\frac{1}{5} \cdot 1\right) = ln\frac{1}{5} =$$

$$= ln\, 1 - ln\, 5 = 0 - ln\, 5 = -ln\, 5 .$$

Esercizio 1269.291b

$$\lim_{x\to 0^+} [ln(tg\, 3x) - ln(5x)] = (-\infty + \infty = ?)$$

$$\lim_{x\to 0^+} [ln(tg\, 3x) - ln(5x)] = \lim_{x\to 0^+} ln\left(\frac{tg\, 3x}{5x}\right) = \lim_{x\to 0^+} ln\left(\frac{3}{5} \cdot \frac{tg\, 3x}{3x}\right) = ln\left(\frac{3}{5} \cdot 1\right) = ln\frac{3}{5} .$$

$$= ln\, 3 - ln\, 5 .$$

Esercizio 1269.292

$$\lim_{x\to +\infty} \frac{ln\left(1 + \frac{1}{5x}\right)}{1 - e^{\frac{3}{x}}} = \left(\frac{0}{0} = ?\right)$$

$Si\ pone \quad \frac{1}{5x} = t \quad \Rightarrow \quad \frac{1}{x} = 5t \qquad\qquad Se\ (x \to +\infty) \quad \Rightarrow \quad (t \to 0)$

$$\lim_{x\to +\infty} \frac{ln\left(1 + \frac{1}{5x}\right)}{1 - e^{\frac{3}{x}}} = \lim_{t\to 0^+} \frac{ln(1 + t)}{1 - e^{3\cdot 5t}} = \lim_{t\to 0^+} \frac{ln(1 + t)}{t} \cdot \frac{15t}{-(e^{15t} - 1)} \cdot \frac{1}{15} = 1 \cdot (-1) \cdot \frac{1}{15} = -\frac{1}{15} .$$

Esercizio 1269.293

$$\lim_{x\to 0} \frac{e^{5+x^2}-e^5}{1-\cos^2 x} = \left(\frac{0}{0}=?\right)$$

$$\lim_{x\to 0} \frac{e^{5+x^2}-e^5}{1-\cos^2 x} = \lim_{x\to 0} \frac{e^5\cdot e^{x^2}-e^5}{\sin^2 x} = \lim_{x\to 0} \frac{e^5\cdot\left(e^{x^2}-1\right)}{\sin^2 x} = \lim_{x\to 0} e^5\cdot\frac{e^{x^2}-1}{x^2}\cdot\frac{x^2}{\sin^2 x} =$$

$$= \lim_{x\to 0} e^5\cdot\frac{e^{x^2}-1}{x^2}\cdot\left(\frac{x}{\sin x}\right)^2 = e^5\cdot 1\cdot 1^2 = e^5 \ .$$

Esercizio 1269.294

$$\lim_{x\to 0} \frac{5\, tg\, x}{e^{\sin x}-\cos x} = \left(\frac{0}{0}=?\right)$$

$$\lim_{x\to 0} \frac{5\, tg\, x}{e^{\sin x}-\cos x} = \lim_{x\to 0} 5\cdot\frac{tg\, x}{x}\cdot\frac{x}{e^{\sin x}-\cos x} = \lim_{x\to 0} 5\cdot\frac{tg\, x}{x}\cdot\frac{1}{\frac{e^{\sin x}-\cos x}{x}} =$$

$$= \lim_{x\to 0} 5\cdot\frac{tg\, x}{x}\cdot\frac{1}{\frac{e^{\sin x}-1+1-\cos x}{x}} = \lim_{x\to 0} 5\cdot\frac{tg\, x}{x}\cdot\frac{1}{\frac{e^{\sin x}-1}{x}+\frac{1-\cos x}{x}} =$$

$$= \lim_{x\to 0} 5\cdot\frac{tg\, x}{x}\cdot\frac{1}{\frac{e^{\sin x}-1}{\sin x}\cdot\frac{\sin x}{x}+\frac{1-\cos x}{x}} =$$

$$= 5\cdot 1\cdot\frac{1}{1\cdot 1+0} = 5 \ .$$

Esercizio 1269.295

$$\lim_{x\to +\infty} \left(1+\frac{x}{2x^2+1}\right)^x = \left(1^{+\infty}=?\right)$$

$$\lim_{x\to +\infty} \left(1+\frac{x}{2x^2+1}\right)^x = \lim_{x\to +\infty} \left(1+\frac{1}{\frac{2x^2+1}{x}}\right)^x = \lim_{x\to +\infty} \left(1+\frac{1}{\frac{2x^2+1}{x}}\right)^{\frac{2x^2+1}{x}\cdot x\cdot\frac{x}{2x^2+1}} =$$

$$= \lim_{x\to +\infty} \left(1+\frac{1}{\frac{2x^2+1}{x}}\right)^{\frac{2x^2+1}{x}\cdot\frac{x^2}{2x^2+1}} = \lim_{x\to +\infty} \left[\left(1+\frac{1}{\frac{2x^2+1}{x}}\right)^{\frac{2x^2+1}{x}}\right]^{\frac{x^2}{2x^2+1}} = e^{\frac{1}{2}} = \sqrt[2]{e} \ .$$

Esercizio 1269.296

$$\lim_{x\to\infty}\left(\frac{3x-1}{3x+2}\right)^{\frac{x}{2}} = (1^{\infty}=?)$$

$$\lim_{x\to\infty}\left(\frac{3x-1}{3x+2}\right)^{\frac{x}{2}} = \lim_{x\to\infty}\left(\frac{3x+2-2-1}{3x+2}\right)^{\frac{x}{2}} = \lim_{x\to\infty}\left(\frac{3x+2}{3x+2}+\frac{-2-1}{3x+2}\right)^{\frac{x}{2}} =$$

$$= \lim_{x\to\infty}\left(1+\frac{-3}{3x+2}\right)^{\frac{x}{2}} =$$

$Si\ pone\ \dfrac{-3}{3x+2}=\dfrac{1}{t} \quad \Rightarrow \quad 3x+2=-3t\,; \quad 3x=-2-3t\,; \quad x=-\dfrac{2}{3}-t$

$Se\ \ (x\to\ +\infty) \quad \Rightarrow \quad (t\to+\infty)$

$$= \lim_{x\to\infty}\left(1+\frac{-3}{3x+2}\right)^{\frac{x}{2}} = \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{\frac{-\frac{2}{3}-t}{2}} = \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{-\frac{1}{3}-\frac{t}{2}} = \lim_{t\to\infty}\frac{\left(1+\frac{1}{t}\right)^{-\frac{1}{3}}}{\left(1+\frac{1}{t}\right)^{\frac{t}{2}}} =$$

$$= \lim_{t\to\infty}\frac{\left(1+\frac{1}{t}\right)^{-\frac{1}{3}}}{\left[\left(1+\frac{1}{t}\right)^{t}\right]^{\frac{1}{2}}} = \frac{1}{e^{\frac{1}{2}}} = \frac{1}{\sqrt[2]{e}}\ .$$