Integrali di funzioni goniometriche

1. $\int sen^2 x \cdot dx$ Dalla formula di duplicazione del coseno si ha: $\cos 2x = \cos^2 x - sen^2 x$ $=$
$= 1 - sen^2 x - sen^2 x = 1 - 2\ sen^2 x$. Pertanto dalla formula $\cos 2x = 1 - 2\ sen^2 x$ si ottiene:
$2\ sen^2 x = 1 - \cos 2x$; $sen^2 x = \frac{1}{2} \cdot (1 - \cos 2x)$. Sostituendo si ha:

$$\int sen^2 x \cdot dx = \int \frac{1}{2} \cdot (1 - \cos 2x) \cdot dx = \frac{1}{2} \cdot \left[\int 1 \cdot dx - \int \cos 2x \cdot dx \right] =$$

$$\frac{1}{2} \cdot \left[\int 1 \cdot dx - \frac{1}{2} \cdot \int 2 \cdot \cos 2x \cdot dx \right] = \frac{1}{2} \cdot \left[x - \frac{1}{2} sen\ 2x \right] + c = \frac{1}{2} \cdot \left[x - \frac{1}{2} \cdot 2 sen\ x \cdot \cos x \right] + c =$$

$$= \frac{1}{2} \cdot [x - sen\ x \cdot \cos x] + c$$

2. $\int \cos^2 x \cdot dx$ Dalla formula di duplicazione del coseno si ha: $\cos 2x = \cos^2 x - sen^2 x$ $=$
$= \cos^2 x - (1 - \cos^2 x) = -1 + 2\ \cos^2 x$. Pertanto dalla formula $\cos 2x = -1 + 2\ \cos^2 x$ si ottiene: $2\ \cos^2 x = 1 + \cos 2x$; $\cos^2 x = \frac{1}{2} \cdot (1 + \cos 2x)$. Sostituendo si ha:

$$\int \cos^2 x \cdot dx = \int \frac{1}{2} \cdot (1 + \cos 2x) \cdot dx = \frac{1}{2} \cdot \left[\int 1 \cdot dx + \int \cos 2x \cdot dx \right] =$$

$$\frac{1}{2} \cdot \left[\int 1 \cdot dx + \frac{1}{2} \cdot \int 2 \cdot \cos 2x \cdot dx \right] = \frac{1}{2} \cdot \left[x + \frac{1}{2} sen\ 2x \right] + c = \frac{1}{2} \cdot \left[x + \frac{1}{2} \cdot 2 sen\ x \cdot \cos x \right] + c =$$

$$= \frac{1}{2} \cdot [x + sen\ x \cdot \cos x] + c$$

3. $\int \frac{1}{sen^2 x \cdot \cos^2 x} dx$ Ricordando che: $sen^2 x + \cos^2 x = 1$ si ha:

$$\int \frac{sen^2 x + \cos^2 x}{sen^2 x \cdot \cos^2 x} dx = \int \frac{sen^2 x}{sen^2 x \cdot \cos^2 x} dx + \int \frac{\cos^2 x}{sen^2 x \cdot \cos^2 x} dx = \int \frac{1}{\cos^2 x} dx + \int \frac{1}{sen^2 x} dx =$$

$$= tg\ x - cotg\ x + c.$$

4. $\int \frac{1}{senx}dx$ Ricordando che: $sen\ 2x = 2sen\ x \cdot cos\ x$ si ha: $sen\ x = 2 \cdot sen\ \frac{x}{2} \cdot cos\ \frac{x}{2}$

$= \int \frac{1}{2 \cdot sen\ \frac{x}{2} \cdot cos\ \frac{x}{2}} dx$ Applicando la relazione fondamentale $sen^2 x + cos^2\ x = 1$ si ha:

$$= \int \frac{sen^2\ \frac{x}{2} + cos^2\ \frac{x}{2}}{2 \cdot sen\ \frac{x}{2} \cdot cos\ \frac{x}{2}} dx = \int \frac{sen^2\ \frac{x}{2}}{2 \cdot sen\ \frac{x}{2} \cdot cos\ \frac{x}{2}} dx + \int \frac{cos^2\ \frac{x}{2}}{2 \cdot sen\ \frac{x}{2} \cdot cos\ \frac{x}{2}} dx =$$

$$= \frac{1}{2} \cdot \int \frac{sen\ \frac{x}{2}}{cos\ \frac{x}{2}} dx + \frac{1}{2} \cdot \int \frac{cos\ \frac{x}{2}}{sen\ \frac{x}{2}} dx = \frac{1}{2} \cdot (-2) \cdot \int \frac{-\frac{1}{2} sen\ \frac{x}{2}}{cos\ \frac{x}{2}} dx + \frac{1}{2} \cdot 2 \cdot \int \frac{\frac{1}{2} cos\ \frac{x}{2}}{sen\ \frac{x}{2}} dx =$$

$$= -log\ \left| cos\ \frac{x}{2} \right| + log\ \left| sen\ \frac{x}{2} \right| + c = log\ \left| \frac{sen\ x/2}{cos\ x/2} \right| + c = log\ \left| tg\ \frac{x}{2} \right| + c$$

5. $\int \frac{1}{cos\ x} dx$ Ricordando che: $cos\ x = sen\ \left(x + \frac{\pi}{2} \right)$ si ha:

$= \int \frac{1}{sen\ \left(x + \frac{\pi}{2} \right)} dx$ Applicando l'integrale dimostrato precedentemente si ha:

$$= log\ \left| tg\ \frac{x + \frac{\pi}{2}}{2} \right| + c = log\ \left| tg\left(\frac{x}{2} + \frac{\pi}{4} \right) \right| + c$$