

## PRODOTTI NOTEVOLI

**Esercizi svolti dall'allieva Palazzo Maria Teresa classe 1B L. Scientifico  
(A.S. 2014/2015)**

$$\bullet (x^2 + 5)(x^2 - 5) = x^4 - 25$$

$$777) (-5a + b^2)(+5a + b^2) = -25a^2 + b^4 = b^4 - 25a^2$$

$$(3x^2 + y^2)(3x^2 - y^2) = 9x^4 - y^4$$

$$778) \left(\frac{1}{4}x + \frac{1}{2}y\right)\left(-\frac{1}{4}x + \frac{1}{2}y\right) = \frac{1}{16}y^2 - \frac{1}{16}x^2$$

$$(x^3 - 2y^2)(x^3 + 2y^2) = x^6 - 4y^4$$

$$779) (2x^2 - 3y^3)(2x^2 + 3y^3) = 4x^4 - 9y^6$$

$$(x^6 - y^3)(x^6 + y^3) = x^{12} - y^6$$

$$780) \left(\frac{1}{2}x - \frac{1}{3}y\right)\left(\frac{1}{2}x + \frac{1}{3}y\right) = \frac{1}{4}x^2 - \frac{1}{9}y^2$$

$$\left(\frac{4}{3}x^2 - 3\right)\left(\frac{4}{3}x^2 + 3\right) = \frac{16}{9}x^4 - 9$$

$$781) (0,1x - 0,2y)(0,1x + 0,2y) = 0,01x^2 - 0,02y^2$$

$$\left(-\frac{3}{2}x + 2\right)\left(+\frac{3}{2}x + 2\right) = 4 - \frac{9}{4}x^2$$

$$782) (xyz - abe)(xyz + abe) = x^2y^2z^2 - a^2b^2e^2$$

$$(uv^2 + u^2v)(-uv^2 + u^2v) = u^4v^2 - u^2v^4$$

$$783) (x^m + y^m)(x^m - y^m) = x^{2m} - y^{2m}$$

$$784) (x^{m+1} - 1)(x^{m+1} + 1) = x^{2m+2} - 1$$

$$785) (x^{m-1} + 2)(x^{m-1} - 2) = x^{2m-2} - 4$$

$$\bullet (2x + y)(2x - y) = 4x^2 - y^2$$

$$\bullet (5x^2 + y^3)(5x^2 - y^3) = 25x^4 - y^6$$

$$\bullet (a+b) \cdot (a+b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

$$\bullet (x+y) \cdot (x+y) = x^2 + 2xy + y^2$$

$$\bullet (x - 2y)^2 = (x - 2y)(x - 2y) = x^2 - 2xy - 2xy + 4y^2 = x^2 - 4xy + 4y^2$$

$$(a - 3)^2 = (a - 3)(a - 3) = a^2 - 6a + 9$$

$$\bullet (-b+2)^2 = +b^2 + 4 - 4b$$

$$\bullet (2a-5b)^2 = 4a^2 + 25b^2 - 20ab$$

$$\bullet (3x-z)^2 = 9x^2 + z^2 - 6xz$$

$$\bullet (-4a-5b)^2 = 16a^2 + 25b^2 + 40ab$$

$$\bullet (2x^2-3y^3)^2 = 4x^4 + 9y^6 - 12x^2y^3$$

$$\bullet \left(-x^2 + \frac{1}{2}\right)^2 = x^4 + \frac{1}{4} - 2x^3$$

$$\bullet \left(\frac{3}{2}xy^2 - \frac{1}{4}x^3\right)^2 = \frac{9}{4}x^2y^4 + \frac{1}{16}x^6 - \frac{3}{4}x^4y^2$$

$$\bullet \left(\frac{5}{2}a^2 - \frac{2}{3}ab^3\right)^2 = \frac{25}{4}a^4 + \frac{4}{9}a^2b^6 - \frac{10}{3}a^3b^3$$

$$\bullet (2a+b+3e) \cdot (2a+b-3e) =$$

$$= [(2a+b)+3e] \cdot [(2a+b)-3e] = (2a+b)^2 - (3e)^2 =$$

$$= 4a^2 + b^2 + 4ab - 9e^2$$

$$786) (2ab - 3a^2b + ab^2)(2ab - 3a^2b - ab^2) =$$

$$= [(2ab - 3a^2b) + ab^2] \cdot [(2ab - 3a^2b) - ab^2] =$$

$$= (2ab - 3a^2b)^2 - (ab^2)^2 =$$

$$= 4a^2b^2 + 9a^4b^2 - 12a^3b^2 - a^2b^4$$

$$800) (2a+5b)^2 = 4a^2 + 25b^2 + 20ab$$

$$801) (2x+y)^2 = 4x^2 + y^2 + 4xy$$

$$802) (2x+2y)^2 = 4x^2 + 4y^2 + 8xy$$

$$803) (3x^3 - 1)^2 = 9x^6 + 1 - 6x^3$$

$$804) (5a - 2b^2)^2 = 25a^2 + 4b^4 - 20ab^2$$

$$805) (-3a+2b)^2 = 9a^2 + 4b^2 - 12ab$$

$$806) (3a^2 - 1)^2 = 9a^4 + 1 - 6a^2$$

$$807) (-3x - 2y)^2 = 9x^2 + 4y^2 + 12xy$$

$$808) \left(\frac{1}{2}a - \frac{3}{2}b\right)^2 = \frac{1}{4}a^2 + \frac{9}{4}b^2 - \frac{3}{2}ab$$

$$809) \left(-\frac{2}{5}a^3 - \frac{1}{2}b^2\right)^2 = \frac{4}{25}a^6 + \frac{1}{4}b^4 + \frac{2}{5}a^3b^2$$

$$810) \left( \frac{1}{100}x^2 - \frac{13}{10}y \right)^2 = \frac{1}{10000}x^4 + \frac{169}{100}y^2 - \frac{26}{100}x^2y$$

$$811) \left( \frac{2}{3}xy^2 - \frac{1}{3}x^2y \right)^2 = \frac{4}{9}x^2y^4 + \frac{1}{9}x^4y^2 - \frac{4}{9}x^3y^3$$

$$812) \left( -\frac{2}{3}a^2 - 3b^2 \right)^2 = \frac{4}{9}a^4 + 9b^4 + 6a^2b^2$$

$$813) (3x^m - b^2)^2 = 9x^{2m} + b^4 - 6x^mb^2$$

$$814) (a^m + 2b^m)^2 = a^{2m} + 4b^{2m} + 4a^mb^m$$

$$\bullet (a+1)(a-1) = a^2 - 1$$

$$\bullet (x+a)(x-a) = x^2 - a^2$$

$$\bullet (x+a-1)(x+a+1) =$$

$$= [(x+a)-1][(x+a)+1] =$$

$$= (x+a)^2 - (1)^2 =$$

$$= x^2 + a^2 - 1 + 2ax$$

$$\bullet (a+b-c)(a+b+c) =$$

$$= [(a+b)-c][(a+b)+c] =$$

$$= (a+b)^2 - c^2 =$$

$$= a^2 + b^2 + 2ab - c^2$$

$$300) (7x - 4y)^2 = 49x^2 + 16y^2 - 56xy$$

$$301) (2y+x)^2 = 4y^2 + x^2 + 4xy$$

$$302) (x^2 - 3y)^2 = x^4 + 9y^2 - 6x^2y$$

$$303) (4a^2 - 3b)^2 = 16a^4 + 9b^2 - 12a^2b$$

$$304) (-1 + 2y)^2 = 1 + 4y^2 - 4y$$

$$305) (-5x + y)^2 = 25x^2 + y^2 - 10xy$$

$$306) (-2a - 4b)^2 = 4a^2 + 16b^2 + 16ab$$

$$307) (-a - b)^2 = a^2 + b^2 + 2ab$$

$$808) \left(-\frac{1}{4}x - \frac{1}{3}y\right)^2 = \frac{1}{16}x^2 + \frac{1}{9}y^2 + \frac{1}{6}xy$$

$$809) \left(-\frac{1}{4}ab + a^2b^2\right)^2 = \frac{1}{16}a^2b^2 + 2a^2b^2 - \frac{3}{2}a^3b^3$$

$$810) (1,2a^2 - 0,1b)^2 = \left(\frac{6}{5}a^2 - \frac{1}{10}b\right)^2 = \frac{36}{25}a^4 + \frac{1}{100}b^2 - \frac{11}{5}a^2b$$

$$811) \left(\frac{1}{4}a^2 - \frac{1}{2}b^3\right)^2 = \frac{1}{16}a^4 + \frac{1}{4}b^6 - \frac{1}{2}a^2b^3$$

$$812) \left(-\frac{1}{2}a + \frac{2}{3}b\right)^2 = \frac{1}{4}a^2 + \frac{4}{9}b^2 - \frac{1}{3}ab$$

$$813) (a^2 - 2b^m)^2 = a^4 + 4b^{2m} - 4a^2b^m$$

$$814) (1 - x^{2m})^2 = 1 + x^{4m} - 2x^{2m}$$

$$\bullet \widehat{(2a - 3b + 5c)^2} = \\ = 4a^2 + 9b^2 + 25c^2 - 12ab + 20ac - 30bc$$

$$\bullet (2x^2 - 3y + 5z^4)^2 = \\ = 4x^4 + 9y^2 + 25z^8 + 12x^2y + 20x^2z^4 - 30yz^5$$

$$\bullet \left(\frac{3}{4}a - \frac{1}{2}b + \frac{3}{4}c\right)^2 = \\ = \frac{9}{16}a^2 + \frac{1}{4}b^2 + \frac{9}{16}c^2 + \frac{9}{8}ac - \frac{3}{4}ab - \frac{3}{4}bc$$

$$\bullet (3a^3 - 2a^2b + 5)^2 = 9a^6 + 4a^5b + 25 - 12a^5b + 30a^3 - 20a^2b$$

$$816) (2x + 2y + 2t)^2 = 4x^2 + 4y^2 + 4t^2 + 8xt + 8xy + 8yt$$

$$817) (3x + 2y + t)^2 = 9x^2 + 4y^2 + t^2 + 12xy + 6xt + 4yt$$

$$818) (2a - 3b - 2c)^2 = 4a^2 + 9b^2 + 4c^2 - 12ab + 8ac + 12bc$$

$$819) (x^2 + 2y + t)^2 = x^4 + 4y^2 + t^2 + 4x^2y + 4yt + 2x^2t$$

$$820) (2x^2 - 3y + 2t)^2 = 4x^4 + 9y^2 + 4t^2 - 12x^2y - 12yt + 8x^2t$$

$$\bullet (a+2b-3c+4x^2)^2 = a^2 + 4b^2 + 9c^2 + 16x^4 + 4ab - 6ac + 8ax^2 - 12bc +$$

$$+ 16bx^2 - 24cx^2$$

$$\bullet \left(2a - \frac{1}{2}b^2 + \frac{2}{3}c + 8\right)^2 = 4a^2 + \frac{1}{4}b^4 + \frac{4}{9}c^2 + 64 - 2ab^2 + \frac{8}{3}ac + 32a - \frac{2}{3}b^2c - \frac{16}{3}bc^2$$

$$\bullet (x+y+1)^2 = x^2 + y^2 + 1 + 2xy + 2y$$

$$821) (0,5x + 0,25y - 0,75t)^2 =$$

$$= \left(\frac{1}{2}x + \frac{1}{4}y - \frac{3}{4}t\right)^2 = \frac{1}{4}x^2 + \frac{1}{16}y^2 + \frac{9}{16}t^2 + \frac{1}{4}xy - \frac{3}{4}xt - \frac{3}{8}yt$$

$$822) (0,1x + 2,1y + 0,2t)^2 = \left(\frac{1}{10}x + \frac{21}{10}y + \frac{1}{5}t\right)^2 =$$

$$= \frac{1}{100}x^2 + \frac{441}{100}y^2 + \frac{1}{25}t^2 + \cancel{\frac{21}{100}xy} + \cancel{\frac{1}{50}xt} + \cancel{\frac{21}{50}yt}$$

$$831) (3x-2)^2 + (x-1)^2 - (2x+1)^2 - 2x(5x-11) =$$

$$= 9x^2 - 12x + x^2 - 2x - (5x^2 + x + 6x) - 10x^2 + 22x =$$

$$= 4x - 6x^2 + 6x = -6x^2 + 10x =$$

$$833) \left(\frac{1}{2}x + y\right)^2 - \left(\frac{1}{2}x - y\right)^2 - 2xy =$$

$$= \cancel{\frac{1}{4}x^2} + y^2 + xy - \cancel{\frac{1}{4}x^2} - y^2 + xy - 2xy = 0.$$

$xz$	$yz$	$z^2$
$xy$	$y^2$	$\underline{yz}$
$x^2$	$xy$	$\circled{xz}$
$x$	$y$	$z$

Quadrato di un trinomio

$$S = 2 \cdot 2; (x+y+z)^2;$$

$$S = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$$

$$\bullet (2x - 3y)^3 = 8x^3 - 27y^3 - 36x^2y - 54xy^2$$

$$\bullet (-2x^2 + 4y^5)^3 = (-2x^2)^3 + (4y^5)^3 + (3)(-2x^2)^2(4y^5) + 3(-2x^2)(4y^5)^2 = \\ = -8x^6 + 64y^{15} + 48x^4y^5 - 96x^2y^{10}$$

$$\bullet (5x^2 - 3xy)^3 = 125x^6 - 135x^5y^3 - 225x^5y + 135x^4y$$

$$842) (2x + 3y)^2 - [(x-y)^2 + (x+y)^2] - 2x(x+6y) = \\ = 4x^2 + 9y^2 + 12xy - [x^2 + y^2 - 2xy + x^2 + y^2 + 2xy] - 2x^2 - 12xy = \\ = 4x^2 + 9y^2 + 12xy - x^2 - y^2 + x^2 - y^2 - 2x^2 - 12xy = 4y^2$$

$$844) (-a - b - c)^2 - (a+b)^2 - 2c(a+b) =$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ac - a^2 - b^2 - 2ab - 2ac - 2bc = c^2$$

$$846) (2a + 3b - 2c)^2 - (a+b+c)^2 - 10b(a-c) - 3(a^2 - c^2) =$$

$$= 4a^2 + 9b^2 + 4c^2 + 12ab - 12bc - 12ac - a^2 - b^2 - c^2 - 2ab + 2bc + 2ac - 10ab + 10bc - 3ac + 3c^2 = 8b^2 + 6c^2 - 6ac$$

$$848) (-2x + 2y)^3 = -8x^3 + 8y^3 + 24x^2y - 24xy^2$$

$$(2x + 2y)^3 = 8x^3 + 8y^3 + 24x^2y + 24xy^2$$

$$(3a - b)^3 = 27a^3 - b^3 - 27a^2b + 9ab^2$$

$$850) (2x^2 - 1)^3 = 8x^6 - 1 - 12x^4 + 6x^2$$

$$(1 - a^3)^3 = 1 - a^9 - 3a^3 + 3a^6$$

$$(-x^2 - 2)^3 = -x^6 - 8 - 6x^4 - 12x^2$$

$$\bullet (2a^3 - 3b^4)^3 = 8a^9 - 27b^{12} - 36a^6b^4 + 54a^3b^8$$

$$\begin{aligned}
 864) & (x-y)^3(x+y)^3 + 3x^2y^2(x^2-y^2) - (x^3-y^3)(x^3+y^3) \\
 = & (x^3-y^3-3x^2y+3y^2x)(x^3+y^3+3x^2y+3y^2x) + 3x^6y^2 - 3x^2y^6 - x^6 + y^6 = \\
 = & (x^3+3y^2x)^2 + (-y^3-3x^2y)(+y^3+3x^2y) + 3x^6y^2 - 3x^2y^6 - x^6 + y^6 = \\
 = & \cancel{x^6} + 9\cancel{x^2y^4} + 6\cancel{x^4y^2} + \cancel{y^6} - \cancel{3x^2y^4} - \cancel{3x^2y^4} = 9x^6y^2 + 3x^6y^2 - 3x^2y^6 + y^6 =
 \end{aligned}$$

$$\begin{aligned}
 865) & (x+2y)^3 - (x-2y)^3 - 4y(x^2+4y^2) = \\
 = & \cancel{x^3} + \cancel{8y^3} + 6x^2y + \cancel{12x^2y} - \cancel{x^3} - \cancel{8y^3} - \cancel{12x^2y} + 6x^2y - 4x^2y - \cancel{16y^3} = 8x^2y
 \end{aligned}$$

$$\begin{aligned}
 866) & (2x-y^2)^3 - (2x-y)^3 + 6xy(2xy+y - y^3 - 2x) = \\
 = & \cancel{8x^3} - y^6 - \cancel{12x^2y^2} + \cancel{6xy^4} - \cancel{8x^3} + y^3 + \cancel{12x^3y} - \cancel{6x^2y^2} + \cancel{12x^2y^2} + \cancel{6xy^2} - \cancel{6xy^4} - \cancel{12x^3} \\
 & - y^6 + y^3
 \end{aligned}$$

### TRIANGOLI DI TARTAGLIA

$$\begin{array}{ccccccc}
 & & 1 & 1 & 1 & & \\
 & & 1 & 1 & 2 & 1 & \\
 & & 1 & 1 & 3 & 3 & 1 \\
 & & 1 & 1 & 5 & 10 & 10 & 5 & 1 \\
 & & 1 & 6 & 15 & 20 & 15 & 6 & 1
 \end{array}$$

$$\begin{aligned}
 867) & \left(\frac{1}{2}a+1\right)^6 = \\
 = & 1\left(\frac{1}{2}a\right)^6(-1)^0 + 6\left(\frac{1}{2}a\right)^5(+1)^1 + 6\left(\frac{1}{2}a\right)^4(+1)^2 + 6\left(\frac{1}{2}a\right)^3(+1)^3 + 1\left(\frac{1}{2}a\right)^2(+1)^4 = \\
 = & \frac{1}{16}a^6 + \frac{1}{2}a^5 + \frac{3}{2}a^4 + 2a
 \end{aligned}$$

$$\begin{aligned}
 868) & (2a-\frac{1}{4}b)^6 = 1(2a)^6 + 6(2a)^5\left(-\frac{1}{4}b\right)^1 + 6(2a)^4\left(-\frac{1}{4}b\right)^2 + 6(2a)^3\left(-\frac{1}{4}b\right)^3 + 1\left(-\frac{1}{4}b\right)^6 = \\
 = & 16a^6 - 8a^5b + \frac{3}{2}a^4b^2 - 8ab^3 - \frac{1}{256}b^6
 \end{aligned}$$

$$\begin{aligned}
 868) & \left(2x+\frac{1}{2}\right)^5 = 1(2x)^5 + 5(2x)^4\left(\frac{1}{2}\right) + 10(2x)^3\left(\frac{1}{2}\right)^2 + 10(2x)^2\left(\frac{1}{2}\right)^3 + 5(2x)\left(\frac{1}{2}\right)^4 + 1\left(\frac{1}{2}\right)^5 = \\
 = & 32x^5 + 40x^4 + 20x^3 + 5x^2 + \frac{5}{8}x + \frac{1}{32}
 \end{aligned}$$

$$868) \left(2x + \frac{1}{3}\right)^6 =$$

$$= 1(2x)^6 + 6(2x)^5\left(\frac{1}{3}\right) + 15(2x)^4\left(\frac{1}{3}\right)^2 + 20(2x)^3\left(\frac{1}{3}\right)^3 + 15(2x)^2\left(\frac{1}{3}\right)^4 + 6(2x)\left(\frac{1}{3}\right)^5 + 1\left(\frac{1}{3}\right)^6$$

$$= 64x^6 + 64x^5 + 80x^4 + \frac{160}{27}x^3 + \frac{20}{27}x^2 + \frac{4}{81}x + \frac{1}{729}$$