Prova di Matematica : Equazioni di grado superiore al II°

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Alunno: ______________________________________ Classe: 2 C

1. *Determina le soluzioni reali e le soluzioni complesse delle seguenti equazioni :*

   $8x^3 + 27 = 0$ $\quad$ $9x^4 - 16 = 0$ $\quad$ $4x^4 - 35x^2 - 9 = 0$ $\quad$ $x^6 - 9x^3 + 8 = 0$

2. *Determina le soluzioni reali delle seguenti equazioni :*

   $8x^3 - 12x^2 - 2x + 3 = 0$ $\quad$ $2x^3 + 13x^2 + 21x + 9 = 0$

   $21x^3 + 37x^2 - 37x - 21 = 0$ $\quad$ $24x^3 - 49x^2 - 49x + 24 = 0$

   $15x^4 - 34x^3 + 34x - 15 = 0$ $\quad$ $72x^5 - 78x^4 - 175x^3 + 175x^2 + 78x - 72 = 0$

3. *Dimostra che l'equazione:* $ax^4 + bx^3 - bx - a = 0$ *è equivalente all'equazione:*

   $(x-1)(x+1)(ax^2 + bx + a) = 0$

Valutazione

Esercizio	1	2	3
Punti	8 + 8 + 10 + 10	8 + 10 + 8 + 8 + 8 + 12	10
Voto	Punteggio grezzo / 10		

Soluzione

$\boxed{8x^3+27=0}$ $\quad (2x+3)(4x^2-6x+9)=0\,;\quad \begin{matrix} 2x+3=0 \\ 4x^2-6x+9=0 \end{matrix}\quad \boxed{x_1=-\frac{2}{3}}$

$$x_{2,3}=\frac{-\frac{b}{2}\mp\sqrt{\left(\frac{b}{2}\right)^2-ac}}{a}=\frac{3\mp\sqrt{9-4\cdot 9}}{4}=\frac{3\mp\sqrt{-27}}{4}=\boxed{\begin{matrix}\dfrac{3-3\sqrt{3}i}{4}\\ \dfrac{3+3\sqrt{3}i}{4}\end{matrix}}$$

$\boxed{9x^4-16=0}$ $\quad (3x^2+4)(3x^2-4)=0\,;\quad \begin{matrix} 3x^2+4=0 \\ 3x^2-4=0 \end{matrix}\quad \begin{matrix} x^2=-\frac{4}{3} \\ x^2=+\frac{4}{3} \end{matrix}\quad \begin{matrix} x_{1,2}=\mp\sqrt{\frac{4}{3}}\,i=\mp\frac{2}{\sqrt{3}}\,i= \\ x_{3,4}=\mp\sqrt{\frac{4}{3}}=\mp\frac{2}{\sqrt{3}}= \end{matrix}\boxed{\begin{matrix}\mp\frac{2}{3}\sqrt{3}\,i\\ \mp\frac{2}{3}\sqrt{3}\end{matrix}}$

$\boxed{4x^4-35x^2-9=0}$ $\quad si\ pone\ x^2=z \quad\rightarrow\quad 4z^2-35z^2-9=0\,;$

$$z_{1,2}=\frac{-b\mp\sqrt{b^2-4ac}}{2a}=\frac{35\mp\sqrt{1225+144}}{2\cdot 4}=\frac{35\mp\sqrt{1369}}{8}=\begin{matrix}\dfrac{35-37}{8}=-\dfrac{2}{8}=-\dfrac{1}{4}\\ \dfrac{35+37}{8}=+\dfrac{72}{8}=+9\end{matrix}$$

$\begin{matrix} x^2=-\frac{1}{4} \\ x^2=+9 \end{matrix}\qquad \begin{matrix} x_{1,2}=\mp\sqrt{-\frac{1}{4}} \\ x_{3,4}=\mp\sqrt{9} \end{matrix}\qquad \boxed{\begin{matrix} x_{1,2}=\mp\frac{1}{2}i \\ x_{3,4}=\mp 3 \end{matrix}}$

$\boxed{x^6-9x^3+8=0}$ $\quad si\ pone\ x^3=z \quad\rightarrow\quad z^2-9z^2+8=0\,;$

$$z_{1,2}=\frac{-b\mp\sqrt{b^2-4ac}}{2a}=\frac{9\mp\sqrt{81-32}}{2\cdot 1}=\frac{9\mp\sqrt{49}}{2}=\begin{matrix}\dfrac{9-7}{2}=\dfrac{2}{2}=1\\ \dfrac{9+7}{2}=\dfrac{16}{2}=8\end{matrix}$$

$\begin{matrix} x^3=1 \\ x^3=8 \end{matrix}\qquad \begin{matrix} x^3-1=0 \\ x^3-8=0 \end{matrix}\qquad \begin{matrix} (x-1)(x^2+x+1)=0 \\ (x-2)(x^2+2x+1)=0 \end{matrix}\qquad \begin{matrix} x-1=0 \\ x^2+x+1=0 \\ x-2=0 \\ x^2+2x+1=0 \end{matrix}\qquad \boxed{\begin{matrix} x=1 \\ x=-1\mp\sqrt{3}\,i \\ x=2 \\ x=\frac{-1\mp\sqrt{3}\,i}{2} \end{matrix}}$

$$8x^3 - 12x^2 - 2x + 3 = 0$$

$4x^2 \cdot (2x-3) - (2x-3) = 0\,;\quad (2x-3)(4x^2-1) = 0\quad \begin{matrix} 2x-3=0 \\ 4x^2-1=0 \end{matrix}$

$x_1 = \frac{3}{2}$

$x^2 = \frac{1}{4} \quad x_{2,3} = \mp\frac{1}{2}$

$$2x^3 + 13x^2 + 21x + 9 = 0$$

	2	13	21	9
$-\frac{3}{2}$		-3	-15	-9
	2	10	6	0

$\left(x+\frac{3}{2}\right)(2x^2+10x+6) = 0 \quad x_1 = -\frac{3}{2}$

$2x^2 + 10x + 6 = 0$

$x^2 + 5x + 3 = 0$

$$x_{2,3} = \frac{-b \mp \sqrt{b^2-4ac}}{2a} = \frac{-5 \mp \sqrt{25-12}}{2\cdot 1} = \frac{-5 \mp \sqrt{13}}{2}$$

$$21x^3 + 37x^2 - 37x - 21 = 0$$

	21	37	-37	-21
1		21	58	+21
	21	58	21	0

$(x-1)(21x^2+58x+21) = 0\,;\quad x-1=0\,;\ x_1 = 1$

$21x^2 + 58x + 21 = 0$

$$x_{2,3} = \frac{-\frac{b}{2} \mp \sqrt{\left(\frac{b}{2}\right)^2 - ac}}{a} = \frac{-29 \mp \sqrt{29^2 - 21\cdot 21}}{21} = \frac{-29 \mp \sqrt{841-441}}{21} = \begin{matrix} \frac{-29-20}{21} = -\frac{49}{21} = -\frac{7}{3} \\ \frac{-29+20}{21} = -\frac{9}{21} = -\frac{3}{7} \end{matrix}$$

$$24x^3 - 49x^2 - 49x + 24 = 0$$

	24	-49	-49	24
-1		-24	73	-24
	24	-73	24	0

$(x+1)(24x^2 - 73x + 24) = 0$; $\quad x + 1 = 0$; $x_1 = -1$

$24x^2 - 73x + 24 = 0$

$$x_{2,3} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} = \frac{73 \mp \sqrt{5329 - 2304}}{2 \cdot 24} = \frac{73 \mp 55}{48} = \begin{cases} \dfrac{73-55}{48} = \dfrac{18}{48} = \dfrac{3}{8} \\ \dfrac{73+55}{48} = \dfrac{128}{48} = \dfrac{8}{3} \end{cases}$$

$$15x^4 - 34x^3 + 34x - 15 = 0$$

	15	-34	0	34	-15
1		15	-19	-19	+15
	15	-19	-19	15	0

$(x-1)(15x^3 - 19x^2 - 19x + 15) = 0$

	15	-19	-19	+15
-1		-15	34	-15
	15	-34	15	0

$(x-1)(x+1)(15x^2 - 34x + 15) = 0 \quad x - 1 = 0$; $x = +1$

$x + 1 = 0$; $x = -1$

$15x^2 - 34x + 15 = 0$

$15x^2 - 34x + 15 = 0$;

$$x_{1,2} = \frac{-\frac{b}{2} \mp \sqrt{\left(\frac{b}{2}\right)^2 - ac}}{a} = \frac{17 \mp \sqrt{(-17)^2 - 15 \cdot 15}}{15} = \frac{17 \mp \sqrt{289 - 225}}{15} = \begin{cases} \dfrac{17-8}{15} = \dfrac{9}{15} = \dfrac{3}{5} \\ \dfrac{17+8}{15} = \dfrac{25}{15} = \dfrac{5}{3} \end{cases}$$

$$72x^5 - 78x^4 - 175x^3 + 175x^2 + 78x - 72 = 0$$

	72	-78	-175	175	78	-72
1		72	-6	-181	-6	+72
	72	-6	-181	-6	72	0

$(x-1)(72x^4 - 6x^3 - 181x^2 - 6x + 72) = 0$ $\qquad x - 1 = 0\,;\ \boxed{x = 1}$

$72x^4 - 6x^3 - 181x^2 - 6x + 72 = 0$

$72x^4 - 6x^3 - 181x^2 - 6x + 72 = 0\,;\qquad 72x^2 - 6x - 181 - \frac{6}{x} + \frac{72}{x} = 0\,;\qquad 72\left(x^2 + \frac{1}{x^2}\right) - 6\left(x + \frac{1}{x}\right) - 181 = 0\,;$

Ponendo: $x + \frac{1}{x} = y$ e $x^2 + \frac{1}{x^2} = y^2 - 2$ si ottiene:

$72(y^2 - 2) - 6y - 181 = 0\,;\qquad 72y^2 - 144 - 6y - 181 = 0\,;\qquad 72y^2 - 6y - 325 = 0\,;$

$$y_{1,2} = \frac{-\frac{b}{2} \mp \sqrt{\left(\frac{b}{2}\right)^2 - ac}}{a} = \frac{3 \mp \sqrt{3^2 + 72 \cdot 325}}{72} = \frac{3 \mp \sqrt{23409}}{72} = \begin{cases} \frac{3-153}{72} = -\frac{150}{72} = -\frac{25}{12} \\ \frac{3+153}{72} = +\frac{156}{72} = \frac{13}{6} \end{cases}$$

Pertanto si ha:

$x + \frac{1}{x} = -\frac{25}{12}\,;\qquad 12x^2 + 12 = -25x\,;\qquad 12x^2 + 25x + 12 = 0\,;$

$$x_{1,2} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} = \frac{-25 \mp \sqrt{25^2 - 4 \cdot 12 \cdot 12}}{2 \cdot 12} = \frac{-25 \mp \sqrt{625 - 576}}{2 \cdot 12} = \frac{-25 \mp \sqrt{49}}{2 \cdot 12} =$$

$$= \begin{cases} \frac{-25-7}{24} = -\frac{32}{24} = \boxed{-\frac{4}{3}} \\ \frac{-25+7}{24} = -\frac{18}{24} = \boxed{-\frac{3}{4}} \end{cases}$$

$x + \frac{1}{x} = \frac{13}{6}\,;\qquad 6x^2 + 6 = 13x\,;\qquad 6x^2 - 13x + 6 = 0\,;$

$$x_{3,4} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} = \frac{13 \mp \sqrt{169 - 144}}{2 \cdot 6} = \frac{13 \mp \sqrt{25}}{12} = \begin{cases} = \frac{13-5}{12} = \frac{8}{12} = \boxed{\frac{2}{3}} \\ = \frac{13+5}{12} = \frac{18}{12} = \boxed{\frac{3}{2}} \end{cases}$$

1. *Dimostra che l'equazione:* $ax^4 + bx^3 - bx - a = 0$ *è equivalente all'equazione:*
   $(x-1)(x+1)(ax^2 + bx + a) = 0$

Il polinomio: $ax^4 + bx^3 - bx - a$ *si può scomporre in fattori:*

$ax^4 + bx^3 - bx - a = a(x^4 - 1) + bx(x^2 - 1) = a(x^2 + 1)(x^2 - 1) + bx(x^2 - 1) =$

$= (x^2 - 1)[a(x^2 + 1) + bx] = (x^2 - 1)[ax^2 + bx + a] = (x + 1)(x - 1)(ax^2 + bx + a)$.

Pertanto l'equazione: $ax^4 + bx^3 - bx - a = 0$ *è equivalente all'equazione:*

$(x-1)(x+1)(ax^2 + bx + a) = 0$.