Liceo Scientifico "G. Galilei" Trebisacce
Anno Scolastico 2011-2012

Prova di Matematica : Frazioni algebriche

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Alunno: _____ Classe: 1 C

A. Scomponi in fattori i seguenti polinomi:

1. $27y^6 + 36y^4z^2 + 12y^2z^4$
2. $x^4 - x^2 + 8x - 16$
3. $\frac{1}{10}a^6 - \frac{3}{5}a^4b^3 + \frac{6}{5}a^2b^6 - \frac{4}{5}b^9$
4. $8b^2 - 8a^2 + a^2x^3y^3 - b^2x^3y^3$
5. $(x-2a)^2 - a^2 - 4x^2 - 4ax$
6. $3a^4 + a^3b^2 - b^2 - 3a$
7. $4x^4 + 16x^3 + 3x^2 - 36x - 27$
8. $6x^4 - 4x^3 + 24x^2 - 16x$
9. $3ac + 3bc - 9c^2 + 3(a+b-3c)^2 - 2a^2 - 2ab + 6ac$
10. $x^5 - x^3y^2 - x^2y^3 + y^5$
11. $4a^2x^4 - 4abx^4 + b^2x^4 - 36a^2y^2 + 36aby^2 - 9b^2y^2$
12. $3z^2 - 22z - 16$

B. Semplifica le seguenti frazioni algebriche:

$\frac{12x - 4 - 9x^2}{2b + 4 - 3bx - 6x}$	⊡ $\frac{3x-2}{b+1}$	⊡ $\frac{3x-2}{b+2}$	⊡ $\frac{3x+2}{b+2}$	⊡ $\frac{3x}{b+2}$
$\frac{x^8 + 2x^4 + 1}{x^{12} + 3x^8 + 3x^4 + 1}$	⊡ $\frac{1}{x^2+1}$	⊡ $\frac{x-1}{x+1}$	⊡ $\frac{1}{x^4-1}$	⊡ $\frac{1}{x^4+1}$
$\frac{a^4 - b^4 + 2a^3b - 2ab^3}{a^4 + 2a^3b + a^2b^2} =$	⊡ $-b^2$	⊡ $\frac{a-b}{a^2}$	⊡ $\frac{a^2-b^2}{a^2}$	⊡ $\frac{a^2-b^2}{a}$
$\frac{a^{2n+1} - a}{a^{n+2} + a^2} =$	⊡ $\frac{a^n-1}{a}$	⊡ $\frac{1-a^n}{a}$	⊡ $\frac{1}{2a}$	⊡ $\frac{a^n-1}{a}$
$\frac{a^2 + 2a^{n+p} - a^{2n} - a^{2p}}{a^{n+p+1} + a^{2n+p} - a^{n+2p}} =$	⊡ $\frac{a+a^n+a^p}{a^{n+p}}$	⊡ $\frac{a-a^n+a^p}{a^{n+p}}$	⊡ $\frac{a+a^n-a^p}{a^{n+p}}$	⊡ $\frac{a-a^n+a^p}{a^n}$

C. Semplifica le seguenti frazioni algebriche:

$$\frac{a^2 + 3a - 1}{a^3 - 3a^2 + 3a - 1} + \frac{a}{a^2 - 2a + 1} + \frac{1}{a-1}$$

$$\left(\frac{b+2}{b-2} - \frac{b+1}{b-1} - \frac{2}{b^2 - 3b + 2}\right) \cdot \frac{2-b}{2}$$

$$\left[\frac{a^3 - a^2}{a^3 + a^2 - 2a} + \frac{2a-1}{a^2 + \frac{3}{2}a - 1}\right]^3 : \left[\frac{a^2 - a}{a^2 - 2a + 1} - \frac{a^2 + 7a}{a^3 + 6a^2 - 7a}\right]^4$$

$$\left\{\left[\left(\frac{2a^2}{8a^4 - a} - \frac{1}{4a^2 + 2a + 1}\right) : \left(\frac{2a^2 - a}{a+1}\right)^{-1} \cdot \frac{12a^3 + 6a^2 + 3a}{3a^2}\right]^{-2} - \frac{4a^4 - 4}{4a^2 - 4}\right\}$$

Valutazione	Esercizio	A	2	3	Totale
	Punti	24	20	36	80

Punti	*0 - 3*	*4 - 8*	*9 - 13*	*14 - 19*	*20 - 25*	*26 - 31*	*32 - 37*	*38 - 43*	*44 - 49*	*50 - 55*	*56 - 61*	*62 - 67*	*68 - 72*	*73 - 76*	*77 - 80*
Voto	*2*	*3*	*3 ½*	*4*	*4 ½*	*5*	*5½*	*6*	*6 ½*	*7*	*7 ½*	*8*	*8 ½*	*9*	*10*

Soluzione

A. Scomponi in fattori i seguenti polinomi:

1. $27y^6 + 36y^4z^2 + 12y^2z^4 = 3y^2(9y^4 + 12y^2z^2 + 4z^4) = 3y^2(3y^2 + 2z^2)^2$.

2. $x^4 - x^2 + 8x - 16 = x^4 - (x-4)^2 = [x^2 + (x-4)][x^2 - (x-4)] = (x^2 + x - 4)(x^2 - x + 4)$.

3. $\frac{1}{10}a^6 - \frac{3}{5}a^4b^3 + \frac{6}{5}a^2b^6 - \frac{4}{5}b^9 = \frac{a^6 - 6a^4b^3 + 12a^2b^6 - 8b^9}{10} = \frac{1}{10}(a^2 - 2b^3)^3$.

4. $8b^2 - 8a^2 + a^2x^3y^3 - b^2x^3y^3 = 8(b^2 - a^2) - x^3y^3(b^2 - a^2) = (b^2 - a^2)(8 - x^3y^3) =$
$= (b + a)(b - a)(2 - xy)(4 + 2xy + x^2y^2)$.

5. $(x - 2a)^2 - a^2 - 4x^2 - 4ax = (x - 2a)^2 - (a^2 + 4x^2 + 4ax) = (x - 2a)^2 - (a + 2x)^2 =$
$= [(x - 2a) + (a + 2x)][(x - 2a) - (a + 2x)] = [x - 2a + a + 2x][x - 2a - a - 2x] =$
$= (3x - a)(-3a - x) = (a - 3x)(3a + x)$.

6. $3a^4 + a^3b^2 - b^2 - 3a = a^3(3a + b^2) - (b^2 + 3a) = (3a + b^2)(a^3 - 1) =$
$= (3a + b^2)(a - 1)(a^2 + a + 1)$.

7. $4x^4 + 16x^3 + 3x^2 - 36x - 27 =$

	4	16	3	-36	-27
-1		-4	-12	+9	+27
	4	12	-9	-27	=

$= (x + 1)(4x^3 + 12x^2 - 9x - 27) =$

	4	12	-9	-27
-3		-12	0	+27
	4	0	-9	=

$= (x + 1)(x + 3)(4x^2 - 9) =$
$= (x + 1)(x + 3)(2x + 3)(2x - 3)$.

8. $6x^4 - 4x^3 + 24x^2 - 16x =$
$= 2x(3x^3 - 2x^2 + 12x - 8) =$

$D_{16} = \{\pm 1; \pm 2; \pm 4; \pm 8; \pm 16\}$
$D_6 = \{\pm 1; \pm 2; \pm 3; \pm 6\}$
$D_{\frac{16}{6}} = \left\{\pm\frac{1}{2}; \pm\frac{1}{3}; \pm\frac{1}{6}; \pm\frac{2}{3}; \ldots\right\}$

$= 2x\left(x - \frac{2}{3}\right)(3x^2 + 12) =$
$= 2x\left(x - \frac{2}{3}\right) \cdot 3(x^2 + 4) =$
$= 2x(3x - 2)(x^2 + 4)$.

	3	-2	+12	-8
$\frac{2}{3}$		+2	0	+8
	3	0	12	=

9. $3ac + 3bc - 9c^2 + 3(a+b-3c)^2 - 2a^2 - 2ab + 6ac =$
$= 3c(a+b-3c) + 3(a+b-3c)^2 - 2a(a+b-3c) =$
$= (a+b-3c)[3c + 3(a+b-3c) - 2a] =$
$= (a+b-3c)(3c + 3a + 3b - 9c - 2a) \;=\; (a+b-3c)(a+3b-6c)\,.$

10. $x^5 - x^3y^2 - x^2y^3 + y^5$
$= x^3(x^2 - y^2) - y^3(x^2 - y^2) =$
$= (x^2 - y^2)(x^3 - y^3) =$
$= (x+y)(x-y)(x-y)(x^2 + xy + y^2) =$
$= (x+y)(x-y)^2(x^2 + xy + y^2)\,.$

11. $4a^2x^4 - 4abx^4 + b^2x^4 - 36a^2y^2 + 36aby^2 - 9b^2y^2 =$
$= x^4(4a^2 - 4ab + b^2) - 9y^2(4a^2 - 4ab + b^2) =$
$= (4a^2 - 4ab + b^2)(x^4 - 9y^2) =$
$= (2a-b)^2(x^2 + 3y)(x^2 - 3y)\,.$

12. $3z^2 - 22z - 16 =$

$p = 3 \cdot (-16) = -48$		$s = -22$
+1	-48	-47
+2	-24	-22

$= 3z^2 + 2z - 24y - 16 =$

$= z(3z+2) - 8(3z+2) =$
$= (3z+2)(z-8)$

B. Semplifica le seguenti frazioni algebriche:

$$\frac{12x - 4 - 9x^2}{2b + 4 - 3bx - 6x} = \qquad C.E.:\ b \neq -2;\ \ x \neq \frac{2}{3}$$
$$= \frac{-(2-3x)^2}{2(b+2) - 3x(b+2)} = \frac{-(2-3x)^2}{(b+2)(2-3x)} = \frac{3x-2}{b+2}\,.$$

$$\frac{x^8 + 2x^4 + 1}{x^{12} + 3x^8 + 3x^4 + 1} \;=\; \frac{(x^4+1)^2}{(x^4+1)^3} \;=\; \frac{1}{x^4+1} \qquad C.E.:\ \forall x \in R$$

$$\frac{a^4 - b^4 + 2a^3b - 2ab^3}{a^4 + 2a^3b + a^2b^2} = \qquad C.E.:\ \ a \neq 0;\ \ a \neq -b$$
$$= \frac{(a^2+b^2)(a^2-b^2) + 2ab(a^2-b^2)}{a^2(a^2 + 2ab + b^2)} = \frac{(a^2-b^2)(a^2+b^2+2ab)}{a^2(a+b)^2} =$$
$$= \frac{(a^2-b^2)(a+b)^2}{a^2(a+b)^2} = \frac{a^2-b^2}{a^2}\,.$$

$$\frac{a^{2n+1}-a}{a^{n+2}+a^2}=$$

$$C.E.:\begin{cases} a\neq 0;\quad a\neq -1 & se\ n\ \text{è}\ dispari \\ a\neq 0; & se\ n\ \text{è}\ pari \end{cases}$$

$$=\frac{a(a^{2n}-1)}{a^2(a^n+1)}=\frac{a(a^n+1)(a^n-1)}{a^2(a^n+1)}=\frac{a^n-1}{a}\ .$$

$$\frac{a^2+2a^{n+p}-a^{2n}-a^{2p}}{a^{n+p+1}+a^{2n+p}-a^{n+2p}}=\qquad C.E.:\quad a\neq 0;\ \dots$$

$$=\frac{a^2-(-2a^{n+p}+a^{2n}+a^{2p})}{a^{n+p}(a+a^n-a^p)}=\frac{a^2-(a^n-a^p)^2}{a^{n+p}(a+a^n-a^p)}=\frac{[a+(a^n-a^p)]\cdot[a-(a^n-a^p)]}{a^{n+p}(a+a^n-a^p)}=$$

$$=\frac{(a+a^n-a^p)\cdot(a-a^n+a^p)}{a^{n+p}(a+a^n-a^p)}=\frac{a-a^n+a^p}{a^{n+p}}\ .$$

C. Semplifica le seguenti frazioni algebriche:

$$\frac{a^2+3a-1}{a^3-3a^2+3a-1}+\frac{a}{a^2-2a+1}+\frac{1}{a-1}=\qquad C.E.:\quad a\neq 1$$

$$=\frac{a^2+3a-1}{(a-1)^3}+\frac{a}{(a-1)^2}+\frac{1}{a-1}=$$

$$=\frac{a^2+3a-1+a(a-1)+(a-1)^2}{(a-1)^3}=$$

$$=\frac{a^2+3a-1+a^2-a+a^2+1-2a}{(a-1)^3}=$$

$$=\frac{3a^2}{(a-1)^3}\ .$$

$$\left(\frac{b+2}{b-2}-\frac{b+1}{b-1}-\frac{2}{b^2-3b+2}\right)\cdot\frac{2-b}{2}=\qquad C.E.:\quad b\neq 1;\quad b\neq 2$$

$$=\left(\frac{b+2}{b-2}-\frac{b+1}{b-1}-\frac{2}{(b-2)(b-1)}\right)\cdot\frac{2-b}{2}=$$

$$=\frac{(b+2)(b-1)-(b-2)(b+1)-2}{(b-2)(b-1)}\cdot\frac{2-b}{2}=$$

$$=\frac{b^2-b+2b-2-b^2-b+2b+2-2}{(b-2)(b-1)}\cdot\frac{2-b}{2}=$$

$$=\frac{2b-2}{(b-2)(b-1)}\cdot\frac{2-b}{2}=$$

$$=\frac{2(b-1)}{(b-2)(b-1)}\cdot\left(-\frac{b-2}{2}\right)\ =\ -1\ .$$

$$\left[\frac{a^3-a^2}{a^3+a^2-2a}+\frac{2a-1}{a^2+\frac{3}{2}a-1}\right]^3:\left[\frac{a^2-a}{a^2-2a+1}-\frac{a^2+7a}{a^3+6a^2-7a}\right]^4=$$

$$C.E.:\ a\neq 0;\ a\neq 1;\ a\neq -2;\ a\neq -7;\ a\neq \frac{1}{2}$$

$$=\left[\frac{a^2(a-1)}{a(a-1)(a+2)}+\frac{2a-1}{\frac{1}{2}(2a^2+3a-2)}\right]^3:\left[\frac{a(a-1)}{(a-1)^2}-\frac{a(a+7)}{a(a-1)(a+7)}\right]^4=$$

$$=\left[\frac{a}{(a+2)}+\frac{2a-1}{\frac{1}{2}(2a-1)(a+2)}\right]^3:\left[\frac{a}{a-1}-\frac{1}{a-1}\right]^4=$$

$$=\left[\frac{a}{(a+2)}+\frac{2(2a-1)}{(2a-1)(a+2)}\right]^3:\left[\frac{a-1}{a-1}\right]^4=$$

$$=\left[\frac{a(2a-1)+2(2a-1)}{(2a-1)(a+2)}\right]^3:\left[\frac{a-1}{a-1}\right]^4=$$

$$=\left[\frac{2a^2-a+4a-2}{(2a-1)(a+2)}\right]^3:1^4=$$

$$=\left[\frac{2a^2+3a-2}{(2a-1)(a+2)}\right]^3:1=$$

$$=\left[\frac{(2a-1)(a+2)}{(2a-1)(a+2)}\right]^3=$$

$$=1^3=$$

$$=1\ .$$

$$\left\{\left[\left(\frac{2a^2}{8a^4-a}-\frac{1}{4a^2+2a+1}\right):\left(\frac{2a^2-a}{a+1}\right)^{-1}\cdot\frac{12a^3+6a^2+3a}{3a^2}\right]^{-2}-\frac{4a^4-4}{4a^2-4}\right\}=$$

$$C.E.:\ a\neq 0;\ a\neq -1;\ a\neq \frac{1}{2}$$

$$=\left\{\left[\left(\frac{2a^2}{a(8a^3-1)}-\frac{1}{4a^2+2a+1}\right):\frac{a+1}{a(2a-1)}\cdot\frac{3a(4a^2+2a+1)}{3a^2}\right]^{-2}-\frac{4(a^2+1)(a^2-1)}{4(a^2-1)}\right\}=$$

$$=\left\{\left[\left(\frac{2a}{(2a-1)(4a^2+2a+1)}-\frac{1}{4a^2+2a+1}\right)\cdot\frac{a(2a-1)}{a+1}\cdot\frac{4a^2+2a+1}{a}\right]^{-2}-(a^2+1)\right\}=$$

$$=\left\{\left[\left(\frac{2a-2a+1}{(2a-1)(4a^2+2a+1)}\right)\cdot\frac{a(2a-1)}{a+1}\cdot\frac{4a^2+2a+1}{a}\right]^{-2}-a^2-1\right\}=$$

$$=\left\{\left[\left(\frac{1}{(2a-1)(4a^2+2a+1)}\right)\cdot\frac{a(2a-1)}{a+1}\cdot\frac{4a^2+2a+1}{a}\right]^{-2}-a^2-1\right\}=$$

$$=\left\{\left[\frac{1}{a+1}\right]^{-2}-a^2-1\right\}=$$

$$=\{(a+1)^2-a^2-1\}=$$

$$=\{a^2+1+2a-a^2-1\}\ =\ 2a\ .$$