Liceo Scientifico "G. Galilei" Trebisacce
Anno Scolastico 2010-2011

Prova di Matematica : Frazioni algebriche

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Alunno: _____ Classe: 1 C

A. Scomponi in fattori i seguenti polinomi:

1. $2x(x-y+z)-3(x-y+z)^2-y(x-y+z)$
2. $x^6-y^6-3x^4y^2+3x^2y^4$
3. $2b^2xy-6cxy+5b^3-15bc$
4. $\frac{1}{9}x^6y^2-2x^4y^3+9x^2y^4$
5. $2xy(a^2-b^2)+b^2x^2-a^2x^2-a^2y^2+b^2y^2$
6. $a^2+14a+49-b^2$
7. $(2x-1)^2-x^2-4x-4$
8. $a^4+5a^3+7a^2+a-2$
9. $(2a^3+16)+(4a^2-16)(a+2)$
10. $2z^2+7z+5$
11. $5a^4-5-3(a^3+a^2+a+1)$
12. $xy^3-x^3y+x^2-y^2$

B. Determina il campo di esistenza delle seguenti frazioni algebriche:

1.	$\frac{2xy}{x^2z\cdot(3x+2)}$	
2.	$\frac{a+5}{(a^2-7a+12)}$	
3.	$\frac{2b+1}{a^2+3a}$	

C. Semplifica le seguenti frazioni algebriche:

$\frac{x^5}{x^2-64}\cdot\frac{x^2-13x+40}{x^3-5x^2}$	☐ $\frac{x^5}{x+8}$	☐ $\frac{x-8}{x+8}$	☐ $\frac{x+5}{x^2\cdot(x-8)}$	☐ $\frac{x^3}{x+8}$
$\frac{a^2-b^2}{ab}-\frac{a^2}{ab+b^2}-\frac{b^2}{a^2-ab}$	☐ $\frac{(a-b)^5}{a^2-b^2}$	☐ $\frac{a^2+2ab-b^2}{a^2-b^2}$	☐ $\frac{a^2-2ab-b^2}{a^2-b^2}$	☐ $\frac{a^2-2ab+b}{a^2-b^2}$
$\frac{a^2-a-6}{a^2-4}\cdot\frac{a^2-3a+2}{a^2-6a+9}$	☐ $\frac{a-1}{a-3}$	☐ $\frac{a+1}{a-3}$	☐ $\frac{a-1}{a+3}$	☐ $\frac{a+1}{a+3}$
$\frac{x^3+x^2}{x+3}\cdot\frac{x-1}{x+4}\cdot\frac{x^2+7x+12}{x^2-1}$	☐ x	☐ x^2	☐ $x-1$	☐ $x-2$
$\left(\frac{y^2}{1-y^2}+\frac{2y}{1-y}\right)\cdot\frac{1-y}{2+3y}$	☐ $\frac{y+1}{y}$	☐ $\frac{y-1}{y}$	☐ $\frac{y}{1+y}$	☐ $\frac{y}{1-y}$

D. Semplifica le seguenti frazioni algebriche:

$$\left(\frac{a+1}{a^3+2a^2-a-2}+\frac{1}{a^2+3a+2}-\frac{2}{a^2+a-2}\right):\left(\frac{1}{a-1}-\frac{1}{a+1}\right)$$

$$\left[\frac{x}{x+y}+1-\frac{2x^2+2xy-y}{(x+y)^2}\right]\cdot\left[\frac{y}{(x+y)^2}+\frac{x}{x+y}-1\right]:\left[\frac{1-(x+y)^2}{(x+y)^4}\right]$$

$$\left(\frac{y-x}{y+x}+\frac{2x^2}{y^2+2xy+x^2}\right):\frac{y^2+x^2}{y^2+xy}+x\cdot\left(\frac{2y}{y^2-x^2}-\frac{1}{y-x}\right)$$

Valutazione	Esercizio	A	B	C	D	Totale
	Punti	2 x 12	4 x 3	4 + 7 + 5 + 5 + 7	12 x 3	100
	Voto	Punteggio grezzo / 10				

Soluzione

A. Scomponi in fattori i seguenti polinomi:

1. $2x(x-y+z)-3(x-y+z)^2-y(x-y+z)=(x-y+z)[2x-3(x-y+z)-y]=$
$=(x-y+z)[2x-3x+3y-3z-y]=(x-y+z)(2y-x-3z)$.

2. $x^6-y^6-3x^4y^2+3x^2y^4=(x^2-y^2)(x^4+x^2y^2+y^4)-3x^2y^2(x^2-y^2)=$
$=(x^2-y^2)(x^4+x^2y^2+y^4-3x^2y^2)=(x^2-y^2)(x^4+y^4-2x^2y^2)=(x^2-y^2)(x^2-y^2)^2=$
$=(x+y)(x-y)(x+y)^2(x-y)^2=(x+y)^3(x-y)^3$.

3. $2b^2xy-6cxy+5b^3-15bc=2xy(b^2-3c)+5b(b^2-3c)=(b^2-3c)(2xy+5b)$

4. $\frac{1}{9}x^6y^2-2x^4y^3+9x^2y^4=x^2y^2\left(\frac{1}{3}x^4-2x^2y+9y^2\right)^2=x^2y^2\left(\frac{1}{3}x^2-3y\right)^2$

5. $2xy(a^2-b^2)+b^2x^2-a^2x^2-a^2y^2+b^2y^2=2xy(a^2-b^2)-x^2(a^2-b^2)-y^2(a^2-b^2)=$
$=(a^2-b^2)(2xy-x^2-y^2)=-(a^2-b^2)(-2xy+x^2+y^2)=(b^2-a^2)(x-y)^2=(b+a)(b-a)(x-y)^2$.

6. $a^2+14a+49-b^2=(a+7)^2-b^2=(a+7+b)(a+7-b)$

7. $(2x-1)^2-x^2-4x-4=(2x-1)^2-(x^2+4x+4)=(2x-1)^2-(x+2)^2=$
$=[(2x-1)+(x+2)]\cdot[(2x-1)-(x+2)]=(3x+1)(x-3)$.

8. $a^4+5a^3+7a^2+a-2=$

	1	5	7	1	-2
-2		-2	-6	-2	+2
	1	3	1	-1	0

$=(a+2)(a^3+3a^2+a-1)=$

	1	3	1	-1
-1		-1	-2	1
	1	2	-1	0

$=(a+2)(a+1)(a^2+2a-1)=$

	1	2	-1
1		1	3
	1	3	≠0

	1	2	-1
-1		-1	-1
	1	1	≠0

$= (a+2)(a+1)(a^2+2a-1)$.

9. $(2a^3+16)+(4a^2-16)(a+2) = 2(a^3+8)+4(a^2-4)(a+2) =$

$= 2(a+2)(a^2-2a+4)+4(a^2-4)(a+2) =$

$= 2(a+2)[a^2-2a+4+2(a^2-4)] =$

$= 2(a+2)[a^2-2a+4+2a^2-8] =$

$= 2(a+2)[3a^2-2a-4] =$

10. $2z^2+7z+5 =$

$= 2z^2+2z+5z+5 = 2z(z+1)+5(z+1) = (z+1)(2z+5)$

$p = 2\cdot 5 = 10$		$s = 7$
1	10	11
2	5	7

11. $5a^4-5-3(a^3+a^2+a+1) =$

$= 5(a^4-1)-3[a^2(a+1)+1(a+1)] =$

$= 5(a^2+1)(a^2-1)-3[(a+1)(a^2+1)] =$

$= 5(a^2+1)(a+1)(a-1)-3[(a+1)(a^2+1)] =$

$= (a^2+1)(a+1)[5(a-1)-3] =$

$= (a^2+1)(a+1)[5a-5-3] =$

$= (a^2+1)(a+1)(5a-8)$.

12. $xy^3-x^3y+x^2-y^2 = xy(y^2-x^2)-1(y^2-x^2) = (y^2-x^2)(xy-1) = (y+x)(y-x)(xy-1)$

B. Determina il campo di esistenza delle seguenti frazioni algebriche:

$$\frac{2xy}{x^2z \cdot (3x+2)}$$

C.E.: $x \neq 0$

$z \neq 0$

$3x + 2 \neq 0$; $3x \neq -2$; $x \neq -\frac{2}{3}$

$$\frac{a+5}{(a^2-7a+12)} = \frac{a+5}{(a-3)(a-4)}$$

C.E.: $a - 3 \neq 0$; $a \neq 3$

$a - 4 \neq 0$; $a \neq 4$

$$\frac{2b+1}{a^2+3a} = \frac{2b+1}{a(a+3)}$$

C.E.: $a \neq 0$

$a + 3 \neq 0$; $a \neq -3$

C. Semplifica le seguenti frazioni algebriche:

$$\frac{x^5}{x^2-64}\cdot\frac{x^2-13x+40}{x^3-5x^2}=$$

C.E.: $x+8\neq 0$; $x\neq -8$

$x-8\neq 0$; $x\neq +8$

$x^2\neq 9$; $x\neq 0$;

$x-5\neq 0$; $x\neq +5$

$$\frac{x^5}{x^2-64}\cdot\frac{x^2-13x+40}{x^3-5x^2}=\frac{x^5}{(x+8)(x-8)}\cdot\frac{(x-5)(x-8)}{x^2(x-5)}=\frac{x^3}{x+8}$$

$$\frac{a^2-b^2}{ab}-\frac{a^2}{ab+b^2}-\frac{b^2}{a^2-ab}=$$

C.E.: $a\neq 0$;

$b\neq 0$;

$a+b\neq 0$; $a\neq -b$

$a-b\neq 0$; $a\neq +b$

$$=\frac{a^2-b^2}{ab}-\frac{a^2}{b(a+b)}-\frac{b^2}{a(a-b)}=$$

$$=\frac{(a^2-b^2)(a^2-b^2)-a^3(a-b)-b^3(a+b)}{ab(a+b)(a-b)}=$$

$$=\frac{a^4+b^4-2a^2b^2-a^4+a^3b-ab^3-b^4}{ab(a+b)(a-b)}=$$

$$=\frac{-2a^2b^2+a^3b-ab^3}{ab(a+b)(a-b)}=$$

$$=\frac{-ab(2ab-a^2+b^2)}{ab(a+b)(a-b)}=$$

$$=\frac{+a^2-b^2-2ab}{(a+b)(a-b)}=$$

$$\frac{a^2-a-6}{a^2-4}\cdot\frac{a^2-3a+2}{a^2-6a+9}=$$

C.E.: $a+2\neq 0\,;\quad a\neq -2$

$a-2\neq 0\,;\quad a\neq +2$

$(a-3)^2\neq 0\,;\quad a-3\neq 0\,;\quad a\neq +3$

$$=\frac{(a+2)(a-3)}{(a+2)(a-2)}\cdot\frac{(a-1)(a-2)}{(a-3)^2}=\frac{a-1}{a-3}$$

$$\frac{x^3+x^2}{x+3}\cdot\frac{x-1}{x+4}\cdot\frac{x^2+7x+12}{x^2-1}=$$

C.E.: $x+3\neq 0\,;\quad x\neq -3$

$x+4\neq 0\,;\quad x\neq -4$

$x+1\neq 0\,;\quad x\neq -1$

$x-1\neq 0\,;\quad x\neq +1$

$$=\frac{x^2(x+1)}{x+3}\cdot\frac{x-1}{x+4}\cdot\frac{(x+3)(x+4)}{(x+1)(x-1)}=$$

$$=x^2$$

$$\left(\frac{y^2}{1-y^2}+\frac{2y}{1-y}\right)\cdot\frac{1-y}{2+3y}=$$

C.E.: $1-y\neq 0\,;\quad y\neq +1$

$1+y\neq 0\,;\quad y\neq -1$

$2+3y\neq 0\,;\quad y\neq -\frac{2}{3}$

$$=\left(\frac{y^2}{(1+y)(1-y)}+\frac{2y}{1-y}\right)\cdot\frac{1-y}{2+3y}=$$

$$=\frac{y^2+2y(1+y)}{(1+y)(1-y)}\cdot\frac{1-y}{2+3y}=$$

$$=\frac{y^2+2y+2y^2}{(1+y)(1-y)}\cdot\frac{1-y}{2+3y}=$$

$$=\frac{3y^2+2y}{(1+y)(1-y)}\cdot\frac{1-y}{2+3y}=$$

$$=\frac{y(3y+2)}{(1+y)(1-y)}\cdot\frac{1-y}{2+3y}=$$

$$=\frac{y}{1+y}$$

D. Semplifica le seguenti frazioni algebriche:

$$\left(\frac{a+1}{a^3+2a^2-a-2}+\frac{1}{a^2+3a+2}-\frac{2}{a^2+a-2}\right):\left(\frac{1}{a-1}-\frac{1}{a+1}\right)=$$

Calcoli:

$a^3+2a^2-a-2=a^2(a+2)-(a+2)=(a+2)(a^2-1)=(a+2)(a+1)(a-1)$

$a^2+3a+2=(a+1)(a+2)$

$a^2+a-2=(a-1)(a+2)$

C.E: $a+2\neq 0\,;\qquad a\neq -2$

$a+1\neq 0\,;\qquad a\neq -1$

$a-1\neq 0\,;\qquad a\neq +1$

$$=\left(\frac{a+1}{(a+2)(a+1)(a-1)}+\frac{1}{(a+1)(a+2)}-\frac{2}{(a-1)(a+2)}\right):\left(\frac{1}{a-1}-\frac{1}{a+1}\right)=$$

$$=\left[\frac{a+1+a-1-2(a+1)}{(a+2)(a+1)(a-1)}\right]:\left[\frac{a+1-(a-1)}{(a+1)(a-1)}\right]=$$

$$=\left[\frac{2a-2a-2}{(a+2)(a+1)(a-1)}\right]:\left[\frac{a+1-a+1}{(a+1)(a-1)}\right]=$$

$$=\left[\frac{-2}{(a+2)(a+1)(a-1)}\right]:\left[\frac{2}{(a+1)(a-1)}\right]=$$

$$=\frac{-2}{(a+2)(a+1)(a-1)}\cdot\frac{(a+1)(a-1)}{2}=$$

$$=-\frac{1}{(a+2)}$$

$$\left[\frac{x}{x+y}+1-\frac{2x^2+2xy-y}{(x+y)^2}\right]\cdot\left[\frac{y}{(x+y)^2}+\frac{x}{x+y}-1\right]:\left[\frac{1-(x+y)^2}{(x+y)^4}\right]=$$

C.E: $x+y\neq 0$; $\quad x\neq -y$

$1-(x+y)^2\neq 0$; $\quad (x+y)^2\neq 1$; $\quad \begin{matrix} x+y\neq -1 \\ x+y\neq +1 \end{matrix}$

$$=\frac{x(x+y)+(x+y)^2-(2x^2+2xy-y)}{(x+y)^2}\cdot\frac{y+x(x+y)-(x+y)^2}{(x+y)^2}:\frac{1-(x+y)^2}{(x+y)^4}=$$

$$=\frac{x^2+xy+x^2+y^2+2xy-2x^2-2xy+y}{(x+y)^2}\cdot\frac{y+x^2+xy-x^2-y^2-2xy}{(x+y)^2}:\frac{1-(x+y)^2}{(x+y)^4}=$$

$$=\frac{y+xy+y^2}{(x+y)^2}\cdot\frac{y-xy-y^2}{(x+y)^2}\cdot\frac{(x+y)^4}{1-(x+y)^2}=$$

$$=\frac{y(1+x+y)}{(x+y)^2}\cdot\frac{y(1-x-y)}{(x+y)^2}\cdot\frac{(x+y)^4}{1-(x+y)^2}=$$

$$=\frac{y(1+x+y)}{(x+y)^2}\cdot\frac{y(1—x-y)}{(x+y)^2}\cdot\frac{(x+y)^4}{\big(1+(x+y)\big)\big(1-(x+y)\big)}=$$

$$=\frac{y(1+x+y)}{(x+y)^2}\cdot\frac{y(1—x-y)}{(x+y)^2}\cdot\frac{(x+y)^4}{(1+x+y)(1-x-y)}=$$

$$=y^2\ .$$

$$\left(\frac{y-x}{y+x}+\frac{2x^2}{y^2+2xy+x^2}\right):\frac{y^2+x^2}{y^2+xy}+x\cdot\left(\frac{2y}{y^2-x^2}-\frac{1}{y-x}\right)=$$

C.E: $y+x\neq 0$; $\quad x\neq -y$

$y-x\neq 0$; $\quad x\neq +y$

$y\neq 0$;

$$=\left(\frac{y-x}{y+x}+\frac{2x^2}{(y+x)^2}\right):\frac{y^2+x^2}{y(y+x)}+x\cdot\left(\frac{2y}{(y+x)(y-x)}-\frac{1}{y-x}\right)=$$

$$=\frac{(y-x)(y+x)+2x^2}{(y+x)^2}\cdot\frac{y(y+x)}{y^2+x^2}+x\cdot\frac{2y-(y+x)}{(y+x)(y-x)}=$$

$$=\frac{y^2-x^2+2x^2}{(y+x)^2}\cdot\frac{y(y+x)}{y^2+x^2}+x\cdot\frac{2y-y-x}{(y+x)(y-x)}=$$

$$=\frac{y^2+x^2}{(y+x)^2}\cdot\frac{y(y+x)}{y^2+x^2}+x\cdot\frac{y-x}{(y+x)(y-x)}=$$

$$=\frac{y^2+x^2}{(y+x)^2}\cdot\frac{y(y+x)}{y^2+x^2}+\frac{x\cdot(y-x)}{(y+x)(y-x)}=$$

$$=\frac{y}{y+x}+\frac{x}{y+x}=$$

$$=\frac{y+x}{y+x}=$$

$$=1\ .$$