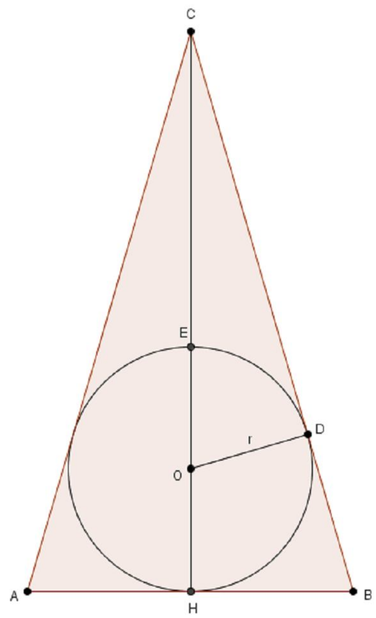


Soluzione 1

Posto $\overline{CO} = x$ con $x \in \left]r, +\infty\right[$ si ha: $\overline{CH} = x + r$ $\overline{CD} = \sqrt{x^2 - r^2}$.

Dai triangoli rettangoli simili $\overset{\Delta}{BCH}$ e $\overset{\Delta}{CDO}$ si ha: $\overline{CH} : \overline{BH} = \overline{CD} : \overline{DO}$;

$(x + r) : \overline{BH} = \sqrt{x^2 - r^2} : r$; $\overline{BH} = \dfrac{(x + r) \cdot r}{\sqrt{x^2 - r^2}}$

La funzione da rendere minima è:

$$S(x) = \frac{1}{2} \cdot \overline{AB} \cdot \overline{CH} = \overline{BH} \cdot \overline{CH} = \frac{(x + r) \cdot r}{\sqrt{x^2 - r^2}} \cdot (x + r); \qquad \boxed{S(x) = r \cdot \frac{(x + r)^2}{\sqrt{x^2 - r^2}}}$$

Agli estremi $x = 0$ e $x = +\infty$ il triangolo degenera in un rettangolo indefinito di area infinita.

$$S'(x) = r \cdot \frac{2 \cdot (x + r) \cdot \sqrt{x^2 - r^2} - (x + r)^2 \cdot \dfrac{2x}{2 \cdot \sqrt{x^2 - r^2}}}{\left(\sqrt{x^2 - r^2}\right)^2} =$$

$$= r \cdot \frac{2 \cdot (x + r) \cdot \sqrt{x^2 - r^2} - \dfrac{(x + r)^2 \cdot x}{\sqrt{x^2 - r^2}}}{x^2 - r^2} = r \cdot \frac{\dfrac{2 \cdot (x + r) \cdot (x^2 - r^2) - (x + r)^2 \cdot x}{\sqrt{x^2 - r^2}}}{x^2 - r^2} =$$

$$= r \cdot \frac{(x + r) \cdot \left[2 \cdot (x^2 - r^2) - (x + r) \cdot x\right]}{(x^2 - r^2) \cdot \sqrt{x^2 - r^2}} = r \cdot \frac{(x + r) \cdot (2x^2 - 2r^2 - x^2 - r\,x)}{(x + r) \cdot (x - r) \cdot \sqrt{x^2 - r^2}} = r \cdot \frac{(x^2 - r\,x - 2r^2)}{(x - r) \cdot \sqrt{x^2 - r^2}} .$$

La derivata prima $S'(x) = 0$ per:

$$r \cdot \frac{(x^2 - r\,x - 2r^2)}{(x - r) \cdot \sqrt{x^2 - r^2}} = 0 ; \quad x^2 - r\,x - 2r^2 = 0 ; \quad x_{1,2} = \frac{r \mp \sqrt{r^2 + 8r^2}}{2} = \frac{r \mp 3r}{2} = \begin{cases} \dfrac{r - 3r}{2} = -r \\ \dfrac{r + 3r}{2} = 2r \end{cases}$$

La soluzione $x = -r$ non è accettabile perché non appartiene all'intervallo $\left]r, +\infty\right[$.

Essendo:

$$S(2r) = r \cdot \frac{(2r + r)^2}{\sqrt{(2r)^2 - r^2}} = r \cdot \frac{9r^2}{\sqrt{3}\,r} = \frac{9}{\sqrt{3}} r^2 = 3\sqrt{3} r^2$$

$$\lim_{x \to r^+} r \cdot \frac{(x + r)^2}{\sqrt{x^2 - r^2}} = +\infty$$

$$\lim_{x \to +\infty} r \cdot \frac{(x + r)^2}{\sqrt{x^2 - r^2}} = +\infty$$

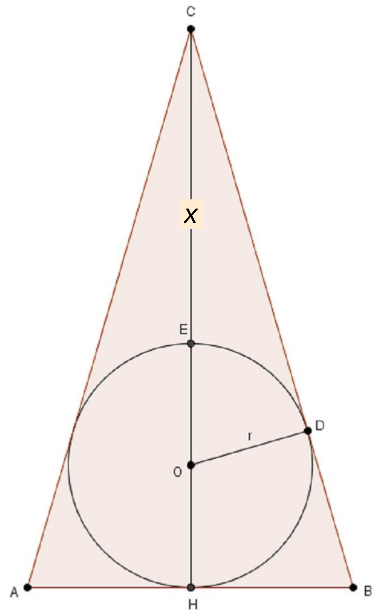
$\Rightarrow$ Il minimo assoluto è $m = 3\sqrt{3} r^2$ assunto nel punto $x = 2r$

Per $x = 2r$ si ha: $\overline{CH} = x + r = 2r + r = 3r$ $\overline{BH} = \dfrac{(x + r) \cdot r}{\sqrt{x^2 - r^2}} = \dfrac{(2r + r) \cdot r}{\sqrt{(2r)^2 - r^2}} = \dfrac{3r^2}{\sqrt{3}\,r} = \sqrt{3}\, r$.

$\overline{AB} = 2\sqrt{3}\, r$ $\overline{BC} = \sqrt{\overline{CH}^2 + \overline{BH}^2} = \sqrt{(3r)^2 + (\sqrt{3}r)^2} = \sqrt{9r^2 + 3r^2} = \sqrt{12r^2} = 2\sqrt{3}\, r$.

Essendo $\overline{AB} = \overline{BC} = \overline{AC} = 2\sqrt{3}\, r$ $\Rightarrow$ il triangolo di area minima è equilatero.

Soluzione 2



Posto $\overline{CE} = x$ con $x \in \left]0, +\infty\right[$ si ha: $\overline{CH} = x + 2r$ e

$$\overline{CD} = \sqrt{(x+r)^2 - r^2} = \sqrt{x^2 + r^2 + 2r\,x - r^2} = \sqrt{x^2 + 2r\,x}$$

Dai triangoli rettangoli simili $\overset{\Delta}{BCH}$ e $\overset{\Delta}{CDO}$ si ha: $\overline{CH} : \overline{BH} = \overline{CD} : \overline{DO}$;

$$(x+2r) : \overline{BH} = \sqrt{x^2 + 2r\,x} : r \quad ; \quad \overline{BH} = \frac{(x+2r)\cdot r}{\sqrt{x^2+2r\,x}}$$

La funzione da rendere minima è:

$$S(x) = \frac{1}{2}\cdot\overline{AB}\cdot\overline{CH} = \frac{(x+2r)\cdot r}{\sqrt{x^2+2r\,x}}\cdot(x+2r) \quad \Rightarrow \quad \boxed{S(x) = r\cdot\frac{(x+2r)^2}{\sqrt{x\cdot(x+2r)}}}.$$

Agli estremi $x = 0$ e $x = +\infty$ il triangolo degenera in un rettangolo indefinito.

$$S'(x) = r\cdot\frac{2\cdot(x+2r)\cdot\sqrt{x\cdot(x+2r)} - (x+2r)^2\cdot\dfrac{2x+2r}{2\cdot\sqrt{x\cdot(x+2r)}}}{x\cdot(x+2r)} =$$

$$= r\cdot\frac{2\cdot(x+2r)\cdot\sqrt{x\cdot(x+2r)} - \dfrac{(x+2r)^2\cdot(x+r)}{\sqrt{x\cdot(x+2r)}}}{x\cdot(x+2r)} = r\cdot\frac{\dfrac{2\cdot(x+2r)\cdot x\cdot(x+2r) - (x+2r)^2\cdot(x+r)}{\sqrt{x\cdot(x+2r)}}}{x\cdot(x+2r)} =$$

$$= r\cdot\frac{2x\cdot(x+2r)^2 - (x+2r)^2\cdot(x+r)}{x(x+2r)\cdot\sqrt{x\cdot(x+2r)}} = r\cdot\frac{(x+2r)^2\cdot(2x-x-r)}{x(x+2r)\cdot\sqrt{x\cdot(x+2r)}} = r\cdot\frac{(x+2r)\cdot(x-r)}{x\cdot\sqrt{x\cdot(x+2r)}} =$$

$$= r\cdot\frac{(x+2r)\cdot(x-r)}{x\cdot\sqrt{x\cdot(x+2r)}}\cdot\frac{\sqrt{x\cdot(x+2r)}}{\sqrt{x\cdot(x+2r)}} = r\cdot\frac{(x+2r)\cdot(x-r)\cdot\sqrt{x\cdot(x+2r)}}{x\cdot x\cdot(x+2r)} = r\cdot\frac{(x-r)\cdot\sqrt{x\cdot(x+2r)}}{x^2}.$$

$$S'(x) = 0 \; ; \quad r\cdot\frac{(x-r)\cdot\sqrt{x\cdot(x+2r)}}{x^2} = 0 \; ; \quad (x-r)\cdot\sqrt{x\cdot(x+2r)} = 0 \; ; \quad \begin{array}{ll} x-r=0 & x=r \\ x=0 & x=0 \;\notin D_{S(x)} \\ x+2r=0 & x=-2r \;\notin D_{S(x)} \end{array}$$

Essendo:

$S(r) = r\cdot\dfrac{(x+2r)^2}{\sqrt{x\cdot(x+2r)}} \quad r\cdot\dfrac{(r+2r)^2}{\sqrt{r\cdot(r+2r)}} = r\cdot\dfrac{9r^2}{\sqrt{3}\,r} = 3\sqrt{3}r^2$ $\lim_{x\to 0^+} r\cdot\dfrac{(x+2r)^2}{\sqrt{x\cdot(x+2r)}} = +\infty$ $\lim_{x\to +\infty} r\cdot\dfrac{(x+2r)^2}{\sqrt{x\cdot(x+2r)}} = +\infty$	$\Rightarrow$	Il minimo assoluto è $m = 3\sqrt{3}r^2$ assunto nel punto $x = r$

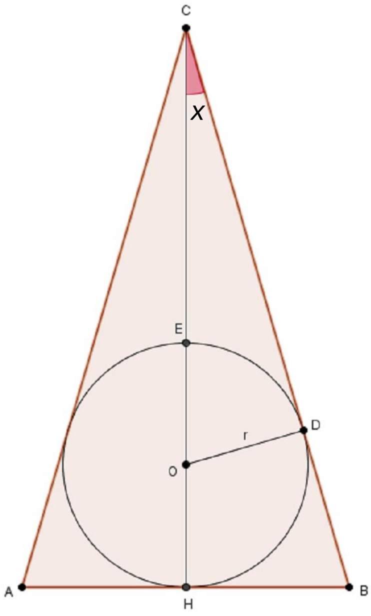
Pertanto: $\overline{CE} = r$; $CH = 3r$; $\overline{AB} = 2\cdot\overline{BH} = 2\cdot\dfrac{(2r+r)\cdot r}{\sqrt{r^2+2r\cdot r}} = 2\cdot\dfrac{3r^2}{\sqrt{3r^2}} = 2\cdot\dfrac{3r}{\sqrt{3}} = 2\sqrt{3}r$.

$$\overline{BC} = \sqrt{\overline{CH}^2 + \overline{BH}^2} = \sqrt{(3r)^2 + \left(\sqrt{3}r\right)^2} = \sqrt{9r^2+3r^2} = \sqrt{12r^2} = 2\sqrt{3}r\;.$$

Avendo dimostrato che $\overline{AB} = \overline{BC} = \overline{AC} = 2\sqrt{3}r$, si può quindi concludere che:

“fra tutti i triangoli isosceli circoscritti a una data circonferenza, il triangolo di area minima è quello equilatero”.

<u>*Soluzione 3*</u>



Posto $B\hat{C}H = x$ con $x \in \left]0, \frac{\pi}{2}\right[$ si ha: $\overline{CO} = \frac{r}{\operatorname{sen} x}$

$$\overline{CH} = r + \frac{r}{\operatorname{sen} x} = r \cdot \frac{1+\operatorname{sen} x}{\operatorname{sen} x} \quad ; \quad \overline{CD} = \frac{r}{\operatorname{tg} x} = r \cdot \frac{\cos x}{\operatorname{sen} x}$$

Dai triangoli simili $\overset{\Delta}{BCH}$ e $\overset{\Delta}{CDO}$ si ha: $\overline{CH} : \overline{BH} = \overline{CD} : \overline{DO}$;

$$r \cdot \frac{1+\operatorname{sen} x}{\operatorname{sen} x} : \overline{BH} = r \cdot \frac{\cos x}{\operatorname{sen} x} : r$$ da cui:

$$\overline{BH} = \frac{r \cdot \frac{1+\operatorname{sen} x}{\operatorname{sen} x} \cdot r}{r \cdot \frac{\cos x}{\operatorname{sen} x}} = r \cdot \frac{1+\operatorname{sen} x}{\operatorname{sen} x} \cdot \frac{\operatorname{sen} x}{\cos x} = r \cdot \frac{1+\operatorname{sen} x}{\cos x} .$$

La funzione da rendere minima è:

$$S(x) = \frac{1}{2} \cdot \overline{AB} \cdot \overline{CH} = \overline{BH} \cdot \overline{CH} = r \cdot \frac{1+\operatorname{sen} x}{\cos x} \cdot r \cdot \frac{1+\operatorname{sen} x}{\operatorname{sen} x}$$

$$S(x) = r^2 \cdot \frac{(1+\operatorname{sen} x)^2}{\operatorname{sen} x \cdot \cos x} .$$

Agli estremi $x = 0$ e $x = +\infty$ il triangolo degenera in un rettangolo indefinito.

$$S'(x) = r^2 \cdot \frac{2 \cdot (1+\operatorname{sen} x) \cdot \cos x \cdot \operatorname{sen} x \cdot \cos x - (1+\operatorname{sen} x)^2 \cdot (\cos^2 x - \operatorname{sen}^2 x)}{\operatorname{sen}^2 x \cdot \cos^2 x} =$$ *Raccogliendo (1+ sen x) si ha:*

$$= r^2 \cdot \frac{(1+\operatorname{sen} x) \cdot \left[2 \cdot \operatorname{sen} x \cdot \cos^2 x - (1+\operatorname{sen} x) \cdot (\cos^2 x - \operatorname{sen}^2 x)\right]}{\operatorname{sen}^2 x \cdot \cos^2 x} =$$ *Trasformando in sen x si ha:*

$$= r^2 \cdot \frac{(1+\operatorname{sen} x) \cdot \left[2 \cdot \operatorname{sen} x \cdot (1-\operatorname{sen}^2 x) - (1+\operatorname{sen} x) \cdot (1 - \operatorname{sen}^2 x - \operatorname{sen}^2 x)\right]}{\operatorname{sen}^2 x \cdot (1-\operatorname{sen}^2 x)} =$$

$$= r^2 \cdot \frac{\cancel{(1+\operatorname{sen} x)} \cdot \left[2\operatorname{sen} x - 2\operatorname{sen}^3 x - (1+\operatorname{sen} x) \cdot (1 - 2\operatorname{sen}^2 x)\right]}{\operatorname{sen}^2 x \cdot \cancel{(1+\operatorname{sen} x)} \cdot (1-\operatorname{sen} x)} =$$

$$= r^2 \cdot \frac{2\operatorname{sen} x - \cancel{2\operatorname{sen}^3 x} - 1 + 2\operatorname{sen}^2 x - \operatorname{sen} x + \cancel{2\operatorname{sen}^3 x}}{\operatorname{sen}^2 x \cdot (1-\operatorname{sen} x)} = r^2 \cdot \frac{2\operatorname{sen}^2 x + \operatorname{sen} x - 1}{\operatorname{sen}^2 x \cdot (1-\operatorname{sen} x)} .$$

$$S'(x) = 0 \;;\quad r^2 \cdot \frac{2\operatorname{sen}^2 x + \operatorname{sen} x - 1}{\operatorname{sen}^2 x \cdot (1-\operatorname{sen} x)} = 0 \;;\quad 2\operatorname{sen}^2 x + \operatorname{sen} x - 1 = 0 \;;\quad \begin{cases} \operatorname{sen} x = -1 & x = \frac{3}{2}\pi \notin D_{S(x)} \\ \operatorname{sen} x = \frac{1}{2} & x = \frac{\pi}{6} \end{cases}$$

Essendo:

$$S\left(\frac{\pi}{6}\right) = r^2 \cdot \frac{\left(1+\operatorname{sen}\frac{\pi}{6}\right)^2}{\operatorname{sen}\frac{\pi}{6} \cdot \cos\frac{\pi}{6}} = r^2 \cdot \frac{\left(1+\frac{1}{2}\right)^2}{\frac{1}{2} \cdot \frac{\sqrt{3}}{2}} = r^2 \cdot \frac{\frac{9}{4}}{\frac{\sqrt{3}}{4}} = 3\sqrt{3}r^2$$

$$\lim_{x\to 0^+} r^2 \cdot \frac{(1+\operatorname{sen} x)^2}{\operatorname{sen} x \cdot \cos x} = +\infty$$

$$\lim_{x\to \left(\frac{\pi}{2}\right)^-} r^2 \cdot \frac{(1+\operatorname{sen} x)^2}{\operatorname{sen} x \cdot \cos x} = +\infty$$

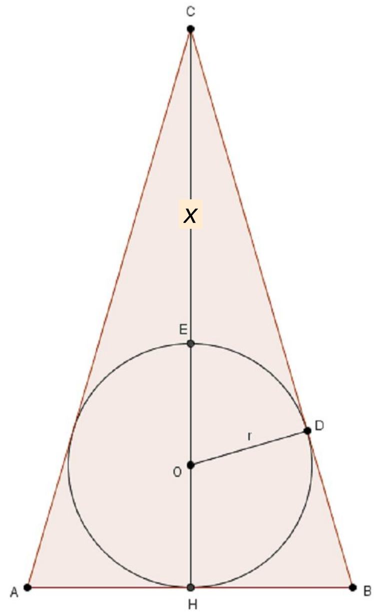
$\Rightarrow$

Il min assoluto è $m = 3\sqrt{3}r^2$

assunto nel punto $x = \frac{\pi}{6}$

Pertanto $\hat{A} = \hat{B} = \hat{C} = \frac{\pi}{3} \Rightarrow$ il triangolo di area minima è equilatero.

Soluzione 4



Posto $\overline{CE} = x$ con $x \in (0, +\infty)$ si ha: $\overline{CH} = x + 2r$ e

$$\overline{CD} = \sqrt{(x+r)^2 - r^2} = \sqrt{x^2 + r^2 + 2r\,x - r^2} = \sqrt{x^2 + 2r\,x}$$

Dai triangoli rettangoli simili $\overset{\Delta}{BCH}$ e $\overset{\Delta}{CDO}$ si ha: $\overline{CH} : \overline{BH} = \overline{CD} : \overline{DO}$;

$$(x+2r) : \overline{BH} = \sqrt{x^2 + 2r\,x} : r \quad ; \quad \overline{BH} = \frac{(x+2r)\cdot r}{\sqrt{x^2+2r\,x}}$$

La funzione da rendere minima è:

$$S(x) = \frac{1}{2}\cdot \overline{AB}\cdot\overline{CH} = \overline{BH}\cdot\overline{CH} = \frac{(x+2r)\cdot r}{\sqrt{x^2+2r\,x}}\cdot(x+2r) = r\cdot\frac{(x+2r)^2}{\sqrt{x\cdot(x+2r)}} .$$

Agli estremi $x = 0$ e $x = +\infty$ il triangolo degenera in un rettangolo indefinito.

$$S'(x) = r\cdot\frac{2\cdot(x+2r)\cdot\sqrt{x\cdot(x+2r)} - (x+2r)^2\cdot\dfrac{2x+2r}{2\cdot\sqrt{x\cdot(x+2r)}}}{x\cdot(x+2r)} =$$

$$= r\cdot\frac{2\cdot(x+2r)\cdot\sqrt{x\cdot(x+2r)} - \dfrac{(x+2r)^2\cdot(x+r)}{\sqrt{x\cdot(x+2r)}}}{x\cdot(x+2r)} = r\cdot\frac{\dfrac{2\cdot(x+2r)\cdot x\cdot(x+2r) - (x+2r)^2\cdot(x+r)}{\sqrt{x\cdot(x+2r)}}}{x\cdot(x+2r)} =$$

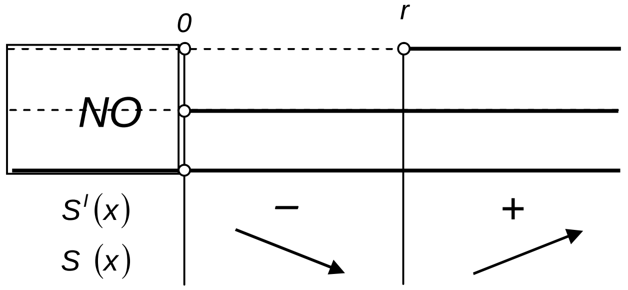
$$= r\cdot\frac{2x\cdot(x+2r)^2 - (x+2r)^2\cdot(x+r)}{x(x+2r)\cdot\sqrt{x\cdot(x+2r)}} = r\cdot\frac{(x+2r)^2\cdot(2x-x-r)}{x(x+2r)\cdot\sqrt{x\cdot(x+2r)}} = r\cdot\frac{(x+2r)\cdot(x-r)}{x\cdot\sqrt{x\cdot(x+2r)}} =$$

$$= r\cdot\frac{(x+2r)\cdot(x-r)}{x\cdot\sqrt{x\cdot(x+2r)}}\cdot\frac{\sqrt{x\cdot(x+2r)}}{\sqrt{x\cdot(x+2r)}} = r\cdot\frac{(x+2r)\cdot(x-r)\cdot\sqrt{x\cdot(x+2r)}}{x\cdot x\cdot(x+2r)} = r\cdot\frac{(x-r)\cdot\sqrt{x\cdot(x+2r)}}{x^2} .$$

$$S'(x) = 0 \;;\quad r\cdot\frac{(x-r)\cdot\sqrt{x\cdot(x+2r)}}{x^2} = 0 \;;\quad (x-r)\cdot\sqrt{x\cdot(x+2r)} = 0 \;;\quad \begin{array}{ll} x - r = 0 & x = r \\ x = 0 & x = 0 \quad \notin D_{S(x)} \\ x + 2r = 0 & x = -2r \quad \notin D_{S(x)} \end{array}$$

$$S'(x) > 0 \;;\quad r\cdot\frac{(x-r)\cdot\sqrt{x\cdot(x+2r)}}{x^2} > 0 \;;$$

$$\begin{array}{ll} x - r > 0 & x > r \\ x\cdot(x+2r) > 0 & x < -2r;\; x > 0 \\ x^2 > 0 & x \neq 0 \end{array}$$



Il punto di minimo si ha per: $x = r$.

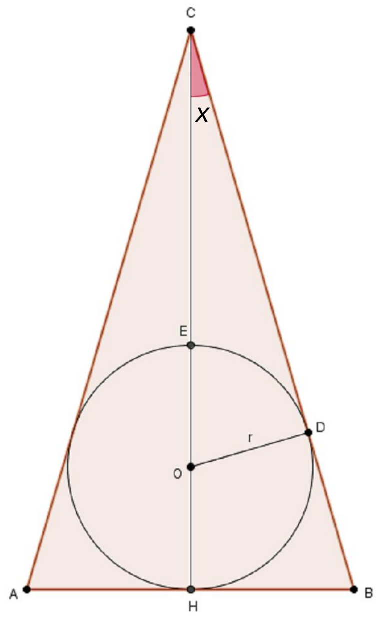
Pertanto: $\overline{CE} = r$; $CH = 3r$; $\overline{AB} = 2\cdot\overline{BH} = 2\cdot\dfrac{(2r+r)\cdot r}{\sqrt{r^2+2r\cdot r}} = 2\cdot\dfrac{3r^2}{\sqrt{3r^2}} = 2\cdot\dfrac{3r}{\sqrt{3}} = 2\sqrt{3}r$.

$$\overline{BC} = \sqrt{\overline{CH}^2 + \overline{BH}^2} = \sqrt{(3r)^2 + (\sqrt{3}r)^2} = \sqrt{9r^2 + 3r^2} = \sqrt{12r^2} = 2\sqrt{3}r .$$

Avendo dimostrato che $\overline{AB} = \overline{BC} = \overline{AC} = 2\sqrt{3}r$, si può quindi concludere che:

fra tutti i triangoli isosceli circoscritti a una data circonferenza, il triangolo di area minima è quello equilatero.

Soluzione 5



Posto $B\hat{C}H = x$ con $x \in \left(0,\ \frac{\pi}{2}\right)$ si ha:

$$\overline{CO} = \frac{r}{\operatorname{sen} x} \ ; \quad \overline{CH} = r + \frac{r}{\operatorname{sen} x} = r \cdot \frac{1+\operatorname{sen} x}{\operatorname{sen} x} \ ; \quad \overline{CD} = \frac{r}{\operatorname{tg} x} = r \cdot \frac{\cos x}{\operatorname{sen} x}$$

Dai triangoli simili $\overset{\Delta}{BCH}$ e $\overset{\Delta}{CDO}$ si ha: $\overline{CH} : \overline{BH} = \overline{CD} : \overline{DO}$;

$$r \cdot \frac{1+\operatorname{sen} x}{\operatorname{sen} x} : \overline{BH} = r \cdot \frac{\cos x}{\operatorname{sen} x} : r \ ;$$

$$\overline{BH} = \frac{r \cdot \frac{1+\operatorname{sen} x}{\operatorname{sen} x} \cdot r}{r \cdot \frac{\cos x}{\operatorname{sen} x}} = r \cdot \frac{1+\operatorname{sen} x}{\operatorname{sen} x} \cdot \frac{\operatorname{sen} x}{\cos x} = r \cdot \frac{1+\operatorname{sen} x}{\cos x} \ .$$

La funzione da rendere minima è:

$$S(x) = \frac{1}{2} \cdot \overline{AB} \cdot \overline{CH} = \overline{BH} \cdot \overline{CH} = r \cdot \frac{1+\operatorname{sen} x}{\cos x} \cdot r \cdot \frac{1+\operatorname{sen} x}{\operatorname{sen} x} = r^2 \cdot \frac{(1+\operatorname{sen} x)^2}{\operatorname{sen} x \cdot \cos x} \ .$$

Agli estremi $x = 0$ e $x = +\infty$ il triangolo degenera in un rettangolo indefinito.

$$S'(x) = r^2 \cdot \frac{2 \cdot (1+\operatorname{sen} x) \cdot \cos x \cdot \operatorname{sen} x \cdot \cos x - (1+\operatorname{sen} x)^2 \cdot (\cos^2 x - \operatorname{sen}^2 x)}{\operatorname{sen}^2 x \cdot \cos^2 x} =$$ *Raccogliendo (1+ sen x) si ha:*

$$= r^2 \cdot \frac{(1+\operatorname{sen} x) \cdot \left[2 \cdot \operatorname{sen} x \cdot \cos^2 x - (1+\operatorname{sen} x) \cdot (\cos^2 x - \operatorname{sen}^2 x)\right]}{\operatorname{sen}^2 x \cdot \cos^2 x} =$$ *Trasformando in sen x si ha:*

$$= r^2 \cdot \frac{(1+\operatorname{sen} x) \cdot \left[2 \cdot \operatorname{sen} x \cdot (1-\operatorname{sen}^2 x) - (1+\operatorname{sen} x) \cdot (1-\operatorname{sen}^2 x - \operatorname{sen}^2 x)\right]}{\operatorname{sen}^2 x \cdot (1-\operatorname{sen}^2 x)} =$$

$$= r^2 \cdot \frac{(1+\operatorname{sen} x) \cdot \left[2\operatorname{sen} x - 2\operatorname{sen}^3 x - (1+\operatorname{sen} x) \cdot (1-2\operatorname{sen}^2 x)\right]}{\operatorname{sen}^2 x \cdot (1+\operatorname{sen} x) \cdot (1-\operatorname{sen} x)} =$$

$$= r^2 \cdot \frac{2\operatorname{sen} x - 2\operatorname{sen}^3 x - 1 + 2\operatorname{sen}^2 x - \operatorname{sen} x + 2\operatorname{sen}^3 x}{\operatorname{sen}^2 x \cdot (1-\operatorname{sen} x)} = r^2 \cdot \frac{2\operatorname{sen}^2 x + \operatorname{sen} x - 1}{\operatorname{sen}^2 x \cdot (1-\operatorname{sen} x)} \ .$$

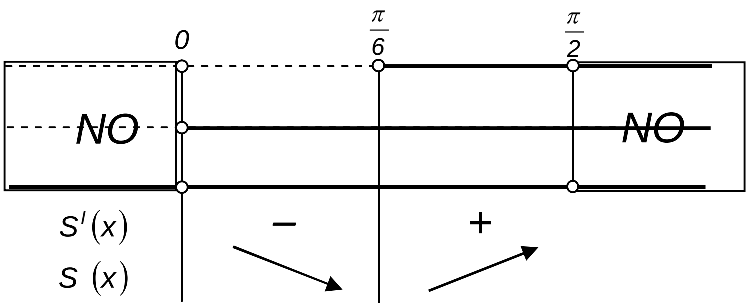
$$S'(x) = 0 \ ; \quad r^2 \cdot \frac{2\operatorname{sen}^2 x + \operatorname{sen} x - 1}{\operatorname{sen}^2 x \cdot (1-\operatorname{sen} x)} = 0 \ ; \quad 2\operatorname{sen}^2 x + \operatorname{sen} x - 1 = 0 \ ; \quad \begin{cases} \operatorname{sen} x = -1 & x = \frac{3}{2}\pi \ \notin D_{S(x)} \\ \operatorname{sen} x = \frac{1}{2} & x = \frac{\pi}{6} \end{cases}$$

$$S'(x) > 0 \ ; \quad r^2 \cdot \frac{2\operatorname{sen}^2 x + \operatorname{sen} x - 1}{\operatorname{sen}^2 x \cdot (1-\operatorname{sen} x)} > 0 \ ;$$

$2\operatorname{sen}^2 x + \operatorname{sen} x - 1 > 0$ $\quad \frac{\pi}{6} < x < \frac{5}{6}\pi$

$\operatorname{sen}^2 x > 0$ $\quad 0 < x < \pi$

$1 - \operatorname{sen} x > 0$ $\quad x \neq \frac{\pi}{2}$



Il punto di minimo si ha per: $x = \frac{\pi}{6} = 30°$ $\Rightarrow$ $B\hat{C}H = 30°$ $\Rightarrow$ $A\hat{C}B = 60°$

Pertanto, fra tutti i triangoli isosceli circoscritti a una data circonferenza, il triangolo di area minima è quello equilatero.