Esercizio x4 – Funzione derivabile

Verificare che la funzione $f(x)=\sqrt[3]{\frac{(x-2)^2}{x+1}}$ non è derivabile in $x_0=2$.

<u>Dimostrazione 1</u>

Il dominio di $f(x)=\sqrt[3]{\frac{(x-2)^2}{x+1}}$ *è* $D\,f=R-\{-1\}$. *In tale Dominio la funzione è continua.*

La derivata prima $f'(x)=\dfrac{1}{3\cdot\sqrt[3]{\left(\frac{(x-2)^2}{x+1}\right)^2}}\cdot\dfrac{2(x-2)^1\cdot 1\cdot(x+1)-(x-2)^2\cdot 1}{(x+1)^2}=$

$$=\frac{1}{3\cdot\sqrt[3]{\frac{(x-2)^4}{(x+1)^2}}}\cdot\frac{(2x-4)\cdot(x+1)-(x-2)^2}{(x+1)^2}=\frac{2x^2+2x-4x-4-x^2-4+4x}{3\cdot(x+1)^2\cdot\sqrt[3]{\frac{(x-2)^4}{(x+1)^2}}}=$$

$$=\frac{x^2+2x-8}{3\cdot(x+1)^2\cdot\sqrt[3]{\frac{(x-2)^4}{(x+1)^2}}}=\frac{(x-2)\cdot(x+4)}{3\cdot(x+1)^2}\cdot\sqrt[3]{\frac{(x+1)^2}{(x-2)^4}}=\frac{(x-2)\cdot(x+4)}{3\cdot(x+1)^2}\cdot\frac{1}{x-2}\cdot\sqrt[3]{\frac{(x+1)^2}{(x-2)}}=$$

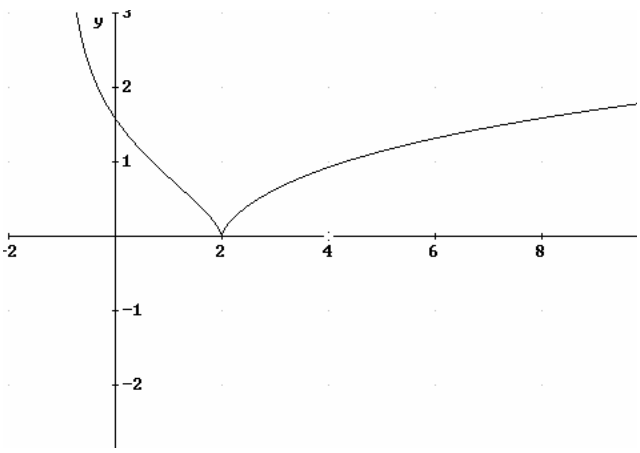
$$=\frac{x+4}{3\cdot(x+1)^2}\cdot\sqrt[3]{\frac{(x+1)^2}{(x-2)}}\,.$$

La derivata prima non è definita in $x_0=2\ \in Dom\ f$.

Inoltre $\lim_{x\to 2^+} f'(x)=\lim_{x\to 2^+}\underbrace{\frac{x+4}{3\cdot(x+1)^2}}_{\frac{2}{9}}\cdot\sqrt[3]{\frac{\overbrace{(x+1)^2}^{9}}{\underbrace{(x-2)}_{0^+}}}=+\infty\,.$

Mentre $\lim_{x\to 2^-} f'(x)=\lim_{x\to 2^-}\underbrace{\frac{x+4}{3\cdot(x+1)^2}}_{\frac{2}{9}}\cdot\sqrt[3]{\frac{(x+1)^2}{\underbrace{(x-2)}_{0^-}}}=-\infty\,.$

Ciò vuol dire che la curva ha in $x_0=2$ *una* ***cuspide*** *rivolta verso il basso.*



Dimostrazione 2

$$\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h} = \lim_{h\to 0}\frac{\sqrt[3]{\frac{(2+h-2)^2}{2+h+1}}-\sqrt[3]{\frac{(2-2)^2}{2+1}}}{h} = \lim_{h\to 0}\frac{\sqrt[3]{\frac{h^2}{3+h}}-\sqrt[3]{\frac{(0)^2}{2+1}}}{h} =$$

$$= \lim_{h\to 0}\frac{\sqrt[3]{\frac{h^2}{3+h}}-0}{h} = \lim_{h\to 0}\frac{\sqrt[3]{\frac{h^2}{3+h}}}{h} = \lim_{h\to 0}\frac{\sqrt[3]{\frac{h^3}{h^3}\cdot\frac{h^2}{(3+h)}}}{h} = \lim_{h\to 0}\frac{h\cdot\sqrt[3]{\frac{h^2}{h^3\cdot(3+h)}}}{h} =$$

$$= \lim_{h\to 0}\sqrt[3]{\frac{h^2}{h^3\cdot(3+h)}} = \lim_{h\to 0}\sqrt[3]{\frac{1}{h\cdot(3+h)}} = \begin{cases} \lim\limits_{h\to 0^+}\sqrt[3]{\frac{1}{h\cdot(3+h)}} & = +\infty \\ \lim\limits_{h\to 0^-}\sqrt[3]{\frac{1}{h\cdot(3+h)}} & = -\infty \end{cases}$$

Dimostrazione 3

$$\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0} = \lim_{x\to 2}\frac{\sqrt[3]{\frac{(x-2)^2}{x+1}}-\sqrt[3]{\frac{(2-2)^2}{2+1}}}{x-2} = \lim_{x\to 2}\frac{\sqrt[3]{\frac{(x-2)^2}{x+1}}-0}{x-2} =$$

$$= \lim_{x\to 2}\frac{\sqrt[3]{\frac{(x-2)^2}{x+1}}}{x-2} = \lim_{x\to 2}\sqrt[3]{\frac{(x-2)^2}{(x-2)^3\cdot(x+1)}} = \lim_{x\to 2}\sqrt[3]{\frac{1}{(x-2)\cdot(x+1)}} =$$

$$= \begin{cases} \lim\limits_{x\to 2^+}\sqrt[3]{\frac{1}{\underbrace{(x-2)}_{0^+}\cdot\underbrace{(x+1)}_{3}}} & = +\infty \\ \lim\limits_{x\to 2^-}\sqrt[3]{\frac{1}{\underbrace{(x-2)}_{0^-}\cdot\underbrace{(x+1)}_{3}}} & = -\infty \end{cases}$$