Radicali

Esercizi svolti

A. Semplifica le seguenti espressioni con numeri irrazionali:

1. $\sqrt[5]{49}$ è irriducibile perché $\sqrt[5]{49} = \sqrt[5]{7^2}$ e $M.C.D.(5;2) = 1$
2. $\sqrt[6]{49} = \sqrt[3\cdot 2]{7^2} = \sqrt[3]{7}$
3. $\sqrt[12]{a^8} = \sqrt[3]{a^2}$ $\quad \sqrt[4]{a^8} = a^2$ $\quad \sqrt[2]{a^4} = a^2$
4. $\sqrt[6]{8} = \sqrt[6]{2^3} = \sqrt[6:3]{2^{3:3}} = \sqrt[2]{2}$ $\quad \sqrt[9]{-5^{18}} = -\sqrt[9]{5^{18}} = -5^2 = -25$
5. $\sqrt[3]{x^3} = x$ $\quad \sqrt[10]{a^{15}} = \sqrt[2]{a^3}$, $C.E.:\ a \geq 0$
6. $\sqrt[6]{16} = \sqrt[6]{2^4} = \sqrt[3]{2^2} = \sqrt[3]{4}$ $\quad \sqrt[6]{8} = \sqrt[6]{2^3} = \sqrt[2]{2}$
7. $\sqrt[6]{(-8)^2} = \sqrt[3]{|-8|} = \sqrt[3]{8} = 2$ $\quad$ la scrittura $\sqrt[6]{(-8)^2} = \sqrt[3]{-8}$ è errata, perché $\sqrt[6]{(-8)^2} \geq 0$ mentre $\sqrt[3]{-8} < 0$
8. $\sqrt[6]{(-2)^2} = \sqrt[3]{|-2|} = \sqrt[3]{2}$ $\quad$ la scrittura $\sqrt[6]{(-2)^2} = \sqrt[3]{-2}$ è evidentemente errata.
9. $\sqrt[4]{\left(2-\sqrt{5}\right)^4} = \left|2-\sqrt{5}\right| = -\left(2-\sqrt{5}\right)$ $\quad$ perchè $\quad 2-\sqrt{5} < 0$
10. $\sqrt[6]{8a^3 - 12a^2 + 6a - 1} = \sqrt[6]{(2a-1)^3} = \sqrt[2]{2a-1}$ $\quad$ con $C.E.:\ a \geq \frac{1}{2}$
11. $\sqrt[2]{a^2 - 6a + 9} = \sqrt[2]{(a-3)^2} = |a-3|$
12. $\sqrt[12]{(5a-2)^8} = \sqrt[3]{(5a-2)^2}$
13. $\left(1+\sqrt{2}\right)^2 + \left(1+\sqrt{2}\right)\cdot\sqrt{2} + \left(\sqrt{2}+1\right)\cdot\left(\sqrt{2}-1\right) = 1+2+2\sqrt{2}+\sqrt{2}+2+2-1 = 6+3\sqrt{2}$.

14. $2\sqrt{8} - 3\sqrt{18} + 5\sqrt{12} - \sqrt{200} + \frac{6}{\sqrt{2}} = 2\cdot 2\sqrt{2} - 3\cdot 3\sqrt{2} + 5\cdot 2\sqrt{3} - 10\sqrt{2} + \frac{6}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} =$

$$= 4\sqrt{2} - 9\sqrt{2} + 10\sqrt{3} - 10\sqrt{2} + \frac{6\sqrt{2}}{2} = 4\sqrt{2} - 9\sqrt{2} + 10\sqrt{3} - 10\sqrt{2} + 3\sqrt{2} = 10\sqrt{3} - 12\sqrt{2} .$$

15. $\left(2\sqrt{3}-5\sqrt{2}\right)^2 - \frac{\sqrt{6}+\sqrt{3}}{2\sqrt{2}+2} - \frac{15}{\sqrt{3}}\cdot\left(\sqrt{3}-4\sqrt{2}\right) = 12+50-20\sqrt{6} - \frac{\sqrt{6}+\sqrt{3}}{2\sqrt{2}+2}\cdot\frac{2\sqrt{2}-2}{2\sqrt{2}-2} - 15 + 60\frac{\sqrt{2}}{\sqrt{3}} =$

$$= 62 - 20\sqrt{6} - \frac{2\sqrt{12} - 2\sqrt{6} + 2\sqrt{6} - 2\sqrt{3}}{8-4} - 15 + 60\frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} =$$

$$= 62 - 20\sqrt{6} - \frac{4\sqrt{3} - 2\sqrt{3}}{4} - 15 + 60\frac{\sqrt{6}}{3} = 62 - 20\sqrt{6} - \frac{2\sqrt{3}}{4} - 15 + 20\sqrt{6} = 47 - \frac{\sqrt{3}}{2} .$$

16. $\frac{1}{(3+\sqrt{2})(2+\sqrt{3})(3-\sqrt{2})(2-\sqrt{3})} = \frac{1}{(3+\sqrt{2})(3-\sqrt{2})(2+\sqrt{3})(2-\sqrt{3})} = \frac{1}{(9-2)(4-3)} = \frac{1}{7\cdot 1} = \frac{1}{7}$.

17. $\sqrt[5]{2\sqrt[4]{2}} = \sqrt[5]{\sqrt[4]{2^4\cdot 2}} = \sqrt[5]{\sqrt[4]{2^5}} = \sqrt[20]{2^5} = \sqrt[4]{2}$.

18. $\left[\left(5^{\sqrt[3]{2}}\right)^{\sqrt{2}}\right]^{\sqrt[6]{2}} = 5^{\sqrt[3]{2}\cdot\sqrt{2}\cdot\sqrt[6]{2}} = 5^{\sqrt[6]{2^2}\cdot\sqrt[6]{2^3}\cdot\sqrt[6]{2}} = 5^{\sqrt[6]{2^2\cdot 2^3\cdot 2}} = 5^{\sqrt[6]{2^6}} = 5^2 = 25$.

19. $\sqrt{7+\sqrt{9+\sqrt{4+5180}}} = \sqrt{7+\sqrt{9+\sqrt{5184}}} = \sqrt{7+\sqrt{9+72}} = \sqrt{7+\sqrt{81}} = \sqrt{7+9} = \sqrt{16} = 4$.

20. $\left(1-2\sqrt{3}\right)\cdot\left(1+\sqrt{3}\right) - \left(\sqrt{3}-5\right)^2 = 1+\sqrt{3}-2\sqrt{3}-2\cdot 3-\left(3+25-10\sqrt{3}\right) =$

$= 1+\sqrt{3}-2\sqrt{3}-6-3-25+10\sqrt{3} = -33+9\sqrt{3}$.

21. Verifica che: $\left(\sqrt{10}\right)^2+\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{17+2\sqrt{10}}\right)^2$

$10+5+2+2\sqrt{10}=17+2\sqrt{10}$; $\quad 17+2\sqrt{10}=17+2\sqrt{10}$.

22. $(4\sqrt{3}+\sqrt{2})^2+(\sqrt{3}+3\sqrt{2})\cdot(\sqrt{3}-3\sqrt{2})-15\cdot\frac{\sqrt{3}-3\sqrt{2}}{\sqrt{3}+3\sqrt{2}} = 48+2+8\sqrt{6}+3-18-15\cdot\frac{\sqrt{3}-3\sqrt{2}}{\sqrt{3}+3\sqrt{2}}\cdot\frac{\sqrt{3}-3\sqrt{2}}{\sqrt{3}-3\sqrt{2}} =$

$= 35+8\sqrt{6}-15\cdot\dfrac{3+18-6\sqrt{6}}{3-18} = 35+8\sqrt{6}-15\cdot\dfrac{21-6\sqrt{6}}{-15} = 35+8\sqrt{6}+21-6\sqrt{6} = 56+2\sqrt{6}$.

23. $3\sqrt{18}-2\sqrt{8}+5\sqrt{12}-6\sqrt{32}-4\sqrt{48} = 3\cdot3\sqrt{2}-2\cdot2\sqrt{2}+5\cdot2\sqrt{3}-6\cdot4\sqrt{2}-4\cdot4\sqrt{3}=$

$= 9\sqrt{2}-4\sqrt{2}+10\sqrt{3}-24\sqrt{2}-16\sqrt{3} = -19\sqrt{2}-6\sqrt{3}$.

24. $\frac{2}{3+2\sqrt{6}}-\frac{\sqrt{2}}{2\sqrt{3}}+\frac{\sqrt{2}}{2(\sqrt{3}+2\sqrt{2})} = \frac{2}{3+2\sqrt{6}}\cdot\frac{3-2\sqrt{6}}{3-2\sqrt{6}}-\frac{\sqrt{2}}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}+\frac{\sqrt{2}}{2(\sqrt{3}+2\sqrt{2})}\cdot\frac{\sqrt{3}-2\sqrt{2}}{\sqrt{3}-2\sqrt{2}} =$

$= \dfrac{6-4\sqrt{6}}{9-24}-\dfrac{\sqrt{6}}{6}+\cdot\dfrac{\sqrt{6}-4}{2\cdot(3-8)} = \dfrac{4\sqrt{6}-6}{15}-\dfrac{\sqrt{6}}{6}+\dfrac{4-\sqrt{6}}{10} = \dfrac{8\sqrt{6}-12-5\sqrt{6}+12-3\sqrt{6}}{30} = 0$.

25. $3\sqrt{27}-5\sqrt{32}+2\sqrt{300}-\frac{6}{\sqrt{3}}+\frac{6}{\sqrt{3}-\sqrt{2}} = 3\cdot3\sqrt{3}-5\cdot\sqrt{2^5}+2\sqrt{100}\sqrt{3}-\frac{6}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}+\frac{6}{\sqrt{3}-\sqrt{2}}\cdot\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} =$

$= 9\sqrt{3}-20\sqrt{2}+20\sqrt{3}-\dfrac{6\sqrt{3}}{3}+\dfrac{6\cdot(\sqrt{3}+\sqrt{2})}{3-2} = 9\sqrt{3}-20\sqrt{2}+20\sqrt{3}-2\sqrt{3}+6\sqrt{3}+6\sqrt{2} =$

$= 33\sqrt{3}-14\sqrt{2}$.

26. $\sqrt[3]{(2-\pi)^3} = 2-\pi$.

27. $\sqrt[4]{(2-\pi)^4} = |2-\pi| = -(2-\pi)$ perché $2-\pi<0$.

28. $\sqrt[4]{(\pi-2)^4} = |\pi-2| = +(\pi-2)$ perché $\pi-2>0$.

29. $\sqrt[3]{\left(\sqrt{5}-\sqrt{6}\right)^3} = \sqrt{5}-\sqrt{6}$.

30. $\sqrt[4]{\left(\sqrt{5}-\sqrt{6}\right)^4} = |\sqrt{5}-\sqrt{6}| = -(\sqrt{5}-\sqrt{6})$ perché $\sqrt{5}-\sqrt{6}<0$.

31. $\sqrt[4]{\left(\sqrt{6}-\sqrt{5}\right)^4} = |\sqrt{6}-\sqrt{5}| = +(\sqrt{6}-\sqrt{5})$ perché $\sqrt{6}-\sqrt{5}>0$.

32. $\sqrt{\left(\sqrt{2}-\sqrt{3}\right)^2} = |\sqrt{2}-\sqrt{3}| = -(\sqrt{2}-\sqrt{3})$ perché $\sqrt{2}-\sqrt{3}<0$.

33. $\sqrt{\left(\sqrt{2}-1\right)^2}+\sqrt{\left(\sqrt{2}-\sqrt{3}\right)^2} = \sqrt{2}-1-(\sqrt{2}-\sqrt{3}) = \sqrt{2}-1-\sqrt{2}+\sqrt{3} = -1+\sqrt{3}$

B. Semplifica le seguenti espressioni letterali irrazionali:

34. $\sqrt{16a^4b^2} = 4a^2|b|$

35. $\sqrt{x^2 - y^2}$ radicale irriducibile

36. $\sqrt[6]{a^3 - 9a^2 + 27a - 27} = \sqrt[6]{(a-3)^3} = \sqrt{a-3}$ con la condizione di esistenza $a \geq 3$

37. $\sqrt[12]{\frac{x^3}{x^3-3x^2+3x-1}} = \sqrt[12]{\frac{x^3}{(x-1)^3}} = \sqrt[4]{\frac{x}{x-1}}$ con le C.E.: $\frac{x^3}{(x-1)^3} \geq 0$ ossia $x < 0 \ \vee \ x \geq 1$

38. $\sqrt[4]{a^2 + 4b^2 + 4ab} = \sqrt[4]{(a+b)^2} = \sqrt{|a+b|}$

39. $\sqrt[15]{\frac{x^3+3x^2+3x+1}{x^6}} = \sqrt[15]{\frac{(x+1)^3}{x^6}} = \sqrt[5]{\frac{x+1}{x^2}}$

40. $\left(\sqrt[5]{3-x}\right)^5 = 3 - x$

41. $\left(\sqrt{3-x}\right)^2 = 3 - x$ con $3 - x \geq 0$ ossia $x \leq 3$

42. $\left(\sqrt{3+x}\right)^6 = (3+x)^3$ con $3 + x \geq 0$ ossia $x \geq -3$

43. $\sqrt{4x^2 + 4x + 1} + \sqrt[4]{(x^2 - 6x + 9)^2} = \sqrt{(2x+1)^2} + \sqrt[4]{(x-3)^4} = |2x+1| + |x-3|$.

44. $\sqrt{x+1} + \sqrt{4x+4} + \sqrt{9x+9} = \sqrt{x+1} + \sqrt{4(x+1)} + \sqrt{9(x+1)} = \sqrt{x+1} + 2\sqrt{x+1} + 3\sqrt{x+1} =$
$6\sqrt{x+1}$ $\quad con\ \text{C.E:}\ x \geq 1$

45. $\sqrt{9x^5 - 18x^4} + \sqrt{4x-8} - 3\sqrt{x^3 - 8 - 6x^2 + 12x} = \sqrt{9x^4(x-2)} + \sqrt{4(x-2)} - 3\sqrt{(x-2)^3} =$ C.E: $x \geq 2$
$= 3x^2\sqrt{x-2} + 2\sqrt{x-2} - 3(x-2)\sqrt{x-2} = (3x^2 + 2 - 3x + 6)\sqrt{x-2} = (3x^2 - 3x + 8)\sqrt{x-2}$.

46. $\frac{1}{\sqrt{a+1}-\sqrt{a}} + \frac{1}{\sqrt{a+1}+\sqrt{a}} - \sqrt[4]{16a^2 + 32a + 16} = \frac{\sqrt{a+1}+\sqrt{a}+\sqrt{a+1}-\sqrt{a}}{(\sqrt{a+1}-\sqrt{a})\cdot(\sqrt{a+1}-\sqrt{a})} - \sqrt[4]{16\cdot(a+1)^2} =$
$= \frac{\sqrt{a+1}+\sqrt{a+1}}{\left(\sqrt{a+1}\right)^2 - \left(\sqrt{a}\right)^2} - 2\sqrt{a+1} = \frac{2\sqrt{a+1}}{a+1-a} - 2\sqrt{a+1} = 2\sqrt{a+1} - 2\sqrt{a+1} = 0$ $\quad con\ \text{C.E:} \begin{cases} a \geq 0 \\ a+1 \geq 0 \end{cases} \quad a \geq 0$

47. $\sqrt{\frac{a^2-1}{a^2+a-2}} : \sqrt[3]{\frac{a^2-4}{a+1}} \cdot \sqrt[6]{\frac{a+2}{a^2+2a+1}} = \sqrt{\frac{(a+1)(a-1)}{(a-1)(a+2)}} : \sqrt[3]{\frac{(a+2)(a-2)}{a+1}} \cdot \sqrt[6]{\frac{a+2}{(a+1)^2}} =$
$= \sqrt{\frac{a+1}{a+2}} : \sqrt[3]{\frac{(a+2)(a-2)}{a+1}} \cdot \sqrt[6]{\frac{a+2}{(a+1)^2}} = \sqrt[6]{\frac{(a+1)^3}{(a+2)^3}} : \sqrt[6]{\frac{(a+2)^2(a-2)^2}{(a+1)^2}} \cdot \sqrt[6]{\frac{a+2}{(a+1)^2}} =$
$= \sqrt[6]{\frac{(a+1)^3}{(a+2)^3} \cdot \frac{(a+1)^2}{(a+2)^2(a-2)^2} \cdot \frac{a+2}{(a+1)^2}} = \sqrt[6]{\frac{(a+1)^3}{(a+2)^4 \cdot (a-2)^2}}$
$con\ C.E.: \begin{cases} \frac{(a+1)(a-1)}{(a-1)(a+2)} \geq 0 \\ a+2 \geq 0 \end{cases} \quad \begin{cases} a < 2 \\ a \geq -2 \end{cases} \vee \ -1 < a < 1 \ \vee \ a < 1 \qquad -1 < a < 1 \ \vee \ a < 1$.

48. $\sqrt[20]{\dfrac{(x^2+2x+1)(x^2+6x+9)}{(16x^2-32x+16)}} = \sqrt[20]{\dfrac{(x+1)^2(x+3)^2}{16(x-1)^2}} = \sqrt[10]{\dfrac{|x+1||x+3|}{4|x-1|}}$.

49. $\sqrt[6]{\dfrac{1+2x^2y}{x^4y^2}+1} = \sqrt[6]{\dfrac{1+2x^2y+x^4y^2}{x^4y^2}} = \sqrt[6]{\dfrac{(1+x^2y)^2}{x^4y^2}} = \sqrt[3]{\dfrac{|1+x^2y|}{x^2|y|}} \qquad x\neq 0 \quad y\neq 0$.

50. $\sqrt[3]{\dfrac{a}{b}+\dfrac{b}{a}+2}\cdot\sqrt[4]{\dfrac{a}{b^2}+\dfrac{b}{a^2}+\dfrac{3(a+b)}{ab}}:\sqrt{\dfrac{a^3+b^3+3a^2b+3ab^2}{a^2b^2}} = \sqrt[3]{\dfrac{a^2+b^2+2ab}{ab}}\cdot\sqrt[4]{\dfrac{a^3+b^3+3a^2b+3ab^2}{a^2b^2}}\cdot\sqrt{\dfrac{a^2b^2}{(a+b)^3}} =$

$= \sqrt[3]{\dfrac{(a+b)^2}{ab}}\cdot\sqrt[4]{\dfrac{(a+b)^3}{a^2b^2}}\cdot\sqrt{\dfrac{a^2b^2}{(a+b)^3}} = \sqrt[12]{\dfrac{(a+b)^8}{a^4b^4}}\cdot\sqrt[12]{\dfrac{(a+b)^9}{a^6b^6}}\cdot\sqrt[12]{\dfrac{a^{12}b^{12}}{(a+b)^{18}}} =$

$= \sqrt[12]{\dfrac{(a+b)^8}{a^4b^4}\cdot\dfrac{(a+b)^9}{a^6b^6}\cdot\dfrac{a^{12}b^{12}}{(a+b)^{18}}} = \sqrt[12]{\dfrac{a^2b^2}{a+b}} \qquad con\ C.E.:\ a+b\geq 0$

51. $\left(\sqrt{\dfrac{2x-1}{2x+1}}+\sqrt{\dfrac{1}{4x^2-1}}\right):\dfrac{1}{\sqrt{2x-1}}-\dfrac{2x}{\sqrt{2x+1}} = \left(\sqrt{\dfrac{2x-1}{2x+1}}+\sqrt{\dfrac{1}{(2x+1)(2x-1)}}\right)\cdot\sqrt{2x-1}-\dfrac{2x}{\sqrt{2x+1}} =$

$= \sqrt{\dfrac{2x-1}{2x+1}}\cdot\sqrt{2x-1}+\sqrt{\dfrac{1}{(2x+1)(2x-1)}}\cdot\sqrt{2x-1}-\dfrac{2x}{\sqrt{2x+1}} =$

$= \sqrt{\dfrac{(2x-1)^2}{2x+1}}+\sqrt{\dfrac{2x-1}{(2x+1)(2x-1)}}-\dfrac{2x}{\sqrt{2x+1}} = \dfrac{2x-1}{\sqrt{2x+1}}+\sqrt{\dfrac{1}{2x+1}}-\dfrac{2x}{\sqrt{2x+1}} =$

$= \dfrac{2x-1}{\sqrt{2x+1}}+\dfrac{1}{\sqrt{2x+1}}-\dfrac{2x}{\sqrt{2x+1}} = \dfrac{2x-1+1-2x}{\sqrt{2x+1}} = \dfrac{0}{\sqrt{2x+1}} = 0$.

$con\ C.E.:\ \begin{cases}\dfrac{2x-1}{2x+1}\geq 0\\(2x+1)(2x-1)>0\end{cases} \qquad \begin{cases}x<-\dfrac{1}{2}\ \vee\ x\geq +\dfrac{1}{2}\\x<-\dfrac{1}{2}\ \vee\ x>+\dfrac{1}{2}\end{cases} \qquad x<-\dfrac{1}{2}\ \vee\ x>+\dfrac{1}{2}$

52. $\sqrt{\dfrac{x-2}{x-1}}\cdot\sqrt[3]{\dfrac{x-1}{x-2}}:\sqrt[4]{\dfrac{x-2}{x-1}} = \sqrt[12]{\left(\dfrac{x-2}{x-1}\right)^6}\cdot\sqrt[12]{\left(\dfrac{x-1}{x-2}\right)^4}\cdot\sqrt[12]{\left(\dfrac{x-1}{x-2}\right)^3} = \sqrt[12]{\dfrac{(x-2)^6}{(x-1)^6}\cdot\dfrac{(x-1)^4}{(x-2)^4}\cdot\dfrac{(x-1)^3}{(x-2)^3}} = \sqrt[12]{\dfrac{x-1}{x-2}}$.

$con\ C.E.:\ \dfrac{x-2}{x-1}\geq 0 \qquad x<1\ \vee\ x\geq 22$.

53. $\left(\sqrt{x-1}+\sqrt{y}\right)\cdot\left(\sqrt{x-1}-\sqrt{y}\right)+\left(\sqrt[6]{2-y}\right)^6$

Le condizioni di esistenza sono: $\begin{cases}x-1\geq 0\\y\geq 0\\2-y\geq 0\end{cases} \qquad \begin{cases}x\geq 1\\y\geq 0\\y\leq 2\end{cases}$ ossia: $C.E.:\ x\geq 1 \quad \wedge \quad 0\leq y\leq 2$

$\left(\sqrt{x-1}+\sqrt{y}\right)\cdot\left(\sqrt{x-1}-\sqrt{y}\right)+\left(\sqrt[6]{2-y}\right)^6 = x-1-y+2-y = x-2y+1$ con $C.E.:\ x\geq 1 \quad \wedge \quad 0\leq y\leq 2$.

54. $\sqrt[3]{x-5} = \begin{cases}+\sqrt[12]{(x-5)^4} & per\ x\geq 5\\-\sqrt[12]{(x-5)^4} & per\ x<5\end{cases}$

55. $\sqrt[4]{x-5} = \sqrt[12]{(x-5)^3}$ per $x\geq 5$

56. Semplifica la seguente espressione: $\dfrac{\sqrt[5]{a^2\cdot\sqrt[3]{a^2}}}{\sqrt[6]{a^5\cdot\sqrt[4]{a^3}}}$ con $a\geq 0$ sia utilizzando le operazioni e le proprietà dei radicali, sia trasformandola in una espressione con esponenti frazionari. Verifica poi, l'uguaglianza dei due risultati ottenuti.

Soluzione 1

$\dfrac{\sqrt[5]{a^2\cdot\sqrt[3]{a^2}}}{\sqrt[6]{a^5}\cdot\sqrt[4]{a^3}} = \dfrac{\sqrt[5]{\sqrt[3]{a^6\cdot a^2}}}{\sqrt[12]{a^{10}}\cdot\sqrt[12]{a^9}} = \dfrac{\sqrt[5]{\sqrt[3]{a^8}}}{\sqrt[12]{a^{10}\cdot a^9}} = \dfrac{\sqrt[15]{a^8}}{\sqrt[12]{a^{19}}} = \dfrac{\sqrt[60]{a^{32}}}{\sqrt[60]{a^{95}}} = \sqrt[60]{\dfrac{a^{32}}{a^{95}}} = \sqrt[60]{\dfrac{1}{a^{63}}} =$

$= \dfrac{1}{\sqrt[60]{a^{63}}} = \dfrac{1}{a\sqrt[60]{a^3}} = \dfrac{1}{a\sqrt[20]{a}}$.

Soluzione 2

$$\frac{\sqrt[5]{a^2\cdot\sqrt[3]{a^2}}}{\sqrt[6]{a^5}\cdot\sqrt[4]{a^3}} = \frac{\sqrt[5]{a^2\cdot a^{\frac{2}{3}}}}{a^{\frac{5}{6}}\cdot a^{\frac{3}{4}}} = \frac{\left(a^{\frac{8}{3}}\right)^{\frac{1}{5}}}{a^{\frac{5}{6}+\frac{3}{4}}} = \frac{a^{\frac{8}{15}}}{a^{\frac{19}{12}}} = a^{\frac{8}{15}-\frac{19}{12}} = a^{\frac{32-95}{60}} = a^{-\frac{63}{60}} = a^{-\frac{21}{20}}\ .$$

Soluzione 3

$$\frac{1}{a\sqrt[20]{a}} = \frac{1}{a\cdot a^{\frac{1}{20}}} = \frac{1}{a^{\frac{21}{20}}} = a^{-\frac{21}{20}}\ .$$

C. Trasporta, se possibile, uno o più fattori fuori dal segno di radice:

57. $\sqrt[4]{32x^3y^6z^8} = \sqrt[4]{2^5x^3y^6z^8} = 2|y|z^2\sqrt[4]{2x^3y^2}$ con la condizione di esistenza $x \geq 0$.

58. $\sqrt{x^3-8y^3-6x^2y+12xy^2} = \sqrt{(x-2y)^3} = (x-2y)\sqrt{x-2y}$ con la condizione di esistenza $x-2y \geq 0$.

D. Razionalizza i denominatori delle seguenti frazioni :

59. $\frac{5}{2\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{2\sqrt{5}\cdot\sqrt{5}} = \frac{5\sqrt{5}}{2\cdot 5} = \frac{\sqrt{5}}{2}$

60. $\frac{6}{\sqrt[7]{8}} = \frac{6}{\sqrt[7]{2^3}}\cdot\frac{\sqrt[7]{2^4}}{\sqrt[7]{2^4}} = \frac{6\sqrt[7]{2^4}}{\sqrt[7]{2^7}} = \frac{6\sqrt[7]{16}}{2} = 3\sqrt[7]{16}$

61. $\frac{5}{\sqrt{7}-\sqrt{2}} = \frac{5}{\sqrt{7}-\sqrt{2}}\cdot\frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}} = \frac{5\cdot\left(\sqrt{7}+\sqrt{2}\right)}{7-2} = \frac{5\cdot\left(\sqrt{7}+\sqrt{2}\right)}{5} = \sqrt{7}+\sqrt{2}$

62. $\frac{x-5}{\sqrt[3]{x}-\sqrt[3]{5}} = \frac{x-5}{\sqrt[3]{x}-\sqrt[3]{5}}\cdot\frac{\sqrt[3]{x^2}+\sqrt[3]{5x}+\sqrt[3]{5^2}}{\sqrt[3]{x^2}+\sqrt[3]{5x}+\sqrt[3]{5^2}} = \frac{(x-5)\cdot\left(\sqrt[3]{x^2}+\sqrt[3]{5x}+\sqrt[3]{5^2}\right)}{\left(\sqrt[3]{x}\right)^3-\left(\sqrt[3]{5}\right)^3} = \frac{(x-5)\cdot\left(\sqrt[3]{x^2}+\sqrt[3]{5x}+\sqrt[3]{5^2}\right)}{x-5} =$

$= \sqrt[3]{x^2}+\sqrt[3]{5x}+\sqrt[3]{25}$.

E. Trasforma i seguenti radicali doppi in somme di radicali semplici:

63. $\sqrt{7+2\sqrt{6}} = \sqrt{7+\sqrt{4\cdot 6}} = \sqrt{7+\sqrt{24}}$

Essendo $a^2-b = 7^2-24 = 25$ un quadrato perfetto conviene sviluppare il radicale con la formula:

$$\sqrt{a+\sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}} + \sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

$$\sqrt{7+\sqrt{24}} = \sqrt{\frac{7+\sqrt{25}}{2}} + \sqrt{\frac{7-\sqrt{25}}{2}} = \sqrt{\frac{7+5}{2}} + \sqrt{\frac{7-5}{2}} = \sqrt{6}+\sqrt{\frac{2}{2}} = \sqrt{6}+1$$

F. Risolvi il seguente sistema lineare:

64. $\begin{cases} \sqrt{2}x + y - 2 = \sqrt{2} \\ x - \frac{y}{2} + 1 = 0 \end{cases}$ $\quad \begin{cases} \sqrt{2}x + y - 2 = \sqrt{2} \\ 2x - y + 2 = 0 \end{cases}$ $\quad \begin{cases} \sqrt{2}x + y - 2 = \sqrt{2} \\ y = 2x + 2 \end{cases}$ $\quad \begin{cases} \sqrt{2}x + 2x + 2 - 2 = \sqrt{2} \\ - \end{cases}$

65. $\begin{cases} (\sqrt{2} + 2)x = \sqrt{2} \\ - \end{cases}$ $\quad \begin{cases} x = \frac{\sqrt{2}}{\sqrt{2}+2} = \frac{\sqrt{2}}{\sqrt{2}+2} \cdot \frac{\sqrt{2}-2}{\sqrt{2}-2} = \frac{2-2\sqrt{2}}{2-4} = \frac{2(1-\sqrt{2})}{-2} = \sqrt{2} - 1 \\ y = 2 \cdot (\sqrt{2} - 1) + 2 = 2\sqrt{2} - 2 + 2 = 2\sqrt{2} \end{cases}$ $\quad \begin{cases} x = \sqrt{2} - 1 \\ y = 2\sqrt{2} \end{cases}$