

SISTEMI LINEARI DI TRE EQUAZIONI IN TRE INCOGNITE
Esercizi svolti dall'allievo Parciante Antonio classe 2A L. Scientifico
(A.S. 2015/2016)

pag. 686, n° 15

$$\begin{cases} x+2y+3z=1 \\ x+y+z=2 \\ 3x+2y+z=3 \end{cases}$$

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 1+6+6 - (9+2+2) = 13-9-2-2 = 0$$

$$\begin{cases} -x+2y+6-3x-3y=1 \\ z=2-x-y \\ 3x+2y+2-x-y=3 \end{cases} \Rightarrow \begin{cases} -2x-y=-5 \\ 2x+y=1 \end{cases} \Rightarrow \begin{cases} 2x+y=5 \\ 2x+y=1 \end{cases}$$

$$\left(\frac{a}{a'}=1\right) = \left(\frac{b}{b'}=1\right) + \left(\frac{c}{c'}=5\right) \quad \text{S. IMPOSS.}$$

H

$$\begin{cases} x+3y-4z=0 \\ 4x-3y+z=0 \\ x+y-z=0 \end{cases}$$

$$D = \begin{vmatrix} 1 & 3 & -4 & 1 & 3 \\ 4 & -3 & 1 & 4 & -3 \\ 1 & 1 & -1 & 1 & 1 \end{vmatrix} = 3+3-16 - (12+1-12) = 3+3-16-1 = -8$$

$$D_x = \begin{vmatrix} 0 & 3 & -4 & 0 & 3 \\ 0 & -3 & 1 & 0 & -3 \\ 0 & 1 & -1 & 0 & 1 \end{vmatrix} = 0+0+0 - (0+0+0) = 0$$

$$D_y = \begin{vmatrix} 1 & 0 & 4 & 1 & 0 \\ 4 & 0 & 1 & 4 & 0 \\ 1 & 0 & -1 & 1 & 0 \end{vmatrix} = 0+0+0 - (0+0+0) = 0$$

$$D_z = \begin{vmatrix} 1 & 3 & 0 & 1 & 3 \\ 4 & -3 & 0 & 4 & -3 \\ 1 & 1 & 0 & 1 & 1 \end{vmatrix} = 0+0+0 - (0+0+0) = 0$$

$$\left(x = \frac{D_x}{D} = \frac{0}{-8} = 0; y = \frac{D_y}{D} = \frac{0}{-8} = 0; z = \frac{D_z}{D} = \frac{0}{-8} = 0 \right)$$

$$\begin{cases} x + \frac{y+2z}{2} = -3 \\ x - \frac{y+2z}{2} = 3 \\ x + \frac{y+3z}{3} = -4 \end{cases} \quad \begin{cases} 2x + y + 2z = -6 \\ 2x - y - 2z = 6 \\ 3x + y + 3z = -12 \end{cases}$$

$$\begin{cases} 2x - 12 - 3x - 3z + 2z = -6 \\ 2x + 12 + 3x + 3z - 2z = 6 \\ y = -12 - 3x - 3z \end{cases} \quad \begin{cases} -x - z = 6 \\ 5x + z = -6 \\ \text{---} \end{cases} \quad \left(\frac{a}{a'} = -\frac{1}{5} \right) \neq \left(\frac{b}{b'} = -1 \right)$$

S. DET.

$$\begin{cases} x = -z - 6 \\ -5z - 30 + z = -6 \\ \text{---} \end{cases} \quad \begin{cases} \text{---} \\ -4z = 24 \\ \text{---} \end{cases} \quad \begin{cases} x = 0 \\ y = 6 \\ z = -6 \end{cases}$$

1/1

$$\begin{cases} x + \frac{y+z}{2} = \frac{3}{2} \\ y + \frac{x-2z}{3} = -2 \\ z - \frac{x+y}{2} = 2 \end{cases} \quad \begin{cases} 2x + y + z = 3 \\ 3y + x - 2z = -6 \\ 2z - x - y = 4 \end{cases} \quad \left(\frac{a}{a'} = -5 \right) \neq \left(\frac{b}{b'} = -1 \right)$$

S. DET.

$$\begin{cases} 2x + y = 1 \\ x + 3y - 2z = -6 \\ 2z - x - y = 4 \end{cases} \quad \begin{cases} y = 1 - 2x \\ x + 3(-6x - 2z) = -6 \\ 2z - x - 1 + 2x = 4 \end{cases} \quad \begin{cases} \text{---} \\ -5x - 2z = -3 \\ x + 2z = 5 \end{cases}$$

$$\begin{cases} \text{---} \\ 5x + 2z = 8 \\ x + 2z = 5 \end{cases} \quad \begin{cases} \text{---} \\ 25 - 10z + 2z = 8 \\ x = 5 - 2z \end{cases} \quad \begin{cases} \text{---} \\ -8z = -16 \\ \text{---} \end{cases}$$

$$\begin{cases} x = 1 \\ y = -1 \\ z = 2 \end{cases}$$

pag. 686, n° 13

$$\begin{cases} x - \frac{1}{2}y - z = 2 \\ \frac{1}{3}(x+y) + x = -\frac{2}{3} \\ x + \frac{1}{2}z = -\frac{1}{2} \end{cases} \quad \begin{cases} 2x - y - 2z = 4 \\ x + y + 3x = -2 \\ 2x + z = -1 \end{cases}$$

$$\begin{cases} 2x - y + 4x + z = 4 \\ 4x + y = -2 \\ z = -2x - 1 \end{cases} \quad \begin{cases} 6x - y = 2 \\ 4x + y = -2 \\ - \end{cases} \quad \left(\frac{a}{a'} = \frac{3}{2}\right) \neq \left(\frac{b}{b'} = -1\right)$$

$$\begin{cases} 6x + z + 4x = z \\ y = -2 - 4x \\ - \end{cases} \quad \begin{cases} 10x = 0 \\ - \\ - \end{cases} \quad \begin{cases} x = 0 \\ y = -2 \\ z = -1 \end{cases}$$

$$\begin{cases} x - y + z = -2 \\ y - z + \frac{1}{2}x = 2 \\ \frac{1}{3}(x + 2y) + 2z = 0 \end{cases} \quad \begin{cases} x - y + z = -2 \\ x + 2y - 2z = 4 \\ x + 2y + 6z = 0 \end{cases}$$

$$D = \begin{vmatrix} 1 & -1 & 1 & 1 & -1 \\ 1 & 2 & -2 & 1 & 2 \\ 1 & 2 & 6 & 1 & 2 \end{vmatrix} = 12 + 2 + 2 - (2 - 4 - 6) = \\ = 16 - 2 + 4 + 6 = \\ = 24$$

$$D_x = \begin{vmatrix} -2 & -1 & 1 & -2 & -1 \\ 4 & 2 & -2 & 4 & 2 \\ 0 & 2 & 6 & 0 & 2 \end{vmatrix} = -24 + 0 + 8 - (0 + 8 - 24) = \\ = -24 + 8 - 8 + 24 = 0$$

$$D_y = \begin{vmatrix} 1 & -2 & 1 & 1 & -2 \\ 1 & 4 & -2 & 1 & 4 \\ 1 & 0 & 6 & 1 & 0 \end{vmatrix} = 24 + 4 + 0 - (4 - 0 - 12) = \\ = 24 + 4 - 4 + 12 = \\ = 36$$

$$D_z = \begin{vmatrix} 1 & -1 & -2 & 1 & -1 \\ 1 & 2 & 4 & 1 & 2 \\ 1 & 2 & 0 & 1 & 2 \end{vmatrix} = 0 - 4 - 4 - (-4 + 8 - 0) = \\ = -8 + 4 - 8 = -12$$

$$\left(x = \frac{D_x}{D} = \frac{0}{24} = 0; y = \frac{D_y}{D} = \frac{36}{24} = \frac{3}{2}; z = \frac{D_z}{D} = \frac{-12}{24} = -\frac{1}{2} \right)$$

$$\begin{cases} \frac{x+y-5z}{2} - 3(x-y) = \frac{3}{4} \\ \frac{2x-3y}{2} + z + \frac{1}{3} = 0 \\ \frac{3y+z}{2} + x = \frac{5}{6} \end{cases} \quad \begin{cases} 2x+2y-10z-12x+12y = 3 \\ 6x-9y+6z+2=0 \\ 9y+3z+6x=5 \end{cases}$$

$$\begin{cases} -10x+14y-10z = 3 \\ 6x-9y+6z = -2 \\ 6x+9y+3z = 5 \end{cases} \quad \begin{cases} - \\ - \\ z = \frac{5}{3} - 2x - 3y \end{cases}$$

$$\begin{cases} -10x+14y - \frac{50}{3} + 20x + 30y = 3 \\ 6x-9y+10-12x-18y = -2 \\ - \end{cases}$$

$$\left(\frac{a}{a_1} = 5 \right) \neq \left(\frac{b}{b_1} = \frac{44}{3} \right)$$

S. DET.

$$\begin{cases} -30x+42y-50+60x+90y = 9 \\ -6x-27y = -12 \\ - \end{cases} \quad \begin{cases} 30x+132y = 59 \\ 6x+27y = 12 \end{cases}$$

$$\begin{cases} 60 - \frac{270}{2}y + 132y = 59 \\ x = 2 - \frac{9}{2}y \\ - \end{cases} \quad \begin{cases} 120 - 270y + 264y = 118 \\ - \\ - \end{cases}$$

$$\begin{cases} -6y = -2 \\ - \\ - \end{cases} \quad \begin{cases} x = 2 - \frac{9}{2} \cdot \frac{1}{3} = 2 - \frac{3}{2} = \frac{4-3}{2} = \frac{1}{2} \\ y = \frac{1}{3} \\ z = \frac{5}{3} - 2 \cdot \frac{1}{3} - 3 \cdot \frac{1}{3} = \frac{5-3-3}{3} = -\frac{1}{3} \end{cases}$$

$$\begin{cases} x = \frac{1}{2} \\ y = \frac{1}{3} \\ z = -\frac{1}{3} \end{cases}$$

→

$$\begin{cases} -5x + 20y - 15z = 6 \\ z + 1 + \frac{2x+y}{3} = 0 \\ \frac{2x-y}{4} = \frac{4}{5} + z - y \end{cases} \quad \begin{cases} -5x + 20y - 15z = 6 \\ 3z + 3 + 2x + y = 0 \\ 10x - 5y = 16 + 20z - 20y \end{cases}$$

$$\begin{cases} -5x + 20y - 15z = 6 \\ 2x + y + 3z = -3 \\ 10x + 15y - 20z = 16 \end{cases} \quad \begin{cases} -5x - 60 - 40x - 60z - 15z = 6 \\ y = -3 - 2x - 3z \\ 10x - 45 - 30x - 45z - 20z = 16 \end{cases}$$

$$\begin{cases} -45x - 75z = 66 \\ - \\ -20x - 65z = 61 \end{cases} \quad \left(\frac{a}{a'} = \frac{9}{4} \right) \neq \left(\frac{b}{b'} = \frac{15}{13} \right)$$

S. DET.

$$\begin{cases} x = -\frac{21}{15} - \frac{5}{3}z \\ - \\ \frac{88}{3} + \frac{100}{3}z - 65z = 61 \end{cases} \quad \begin{cases} - \\ - \\ 88 + 100z - 195z = 183 \end{cases}$$

$$\begin{cases} - \\ - \\ -85z = 95 \end{cases} \quad \begin{cases} x = -\frac{22}{15} + \frac{5}{3} = \frac{-22 + 25}{15} = \frac{3}{15} = \frac{1}{5} \\ y = -3 - 2 \cdot \frac{1}{5} + 3 = -3 - \frac{2}{5} + 3 = -\frac{2}{5} \\ z = -1 \end{cases}$$

$$\begin{cases} x = \frac{1}{5} \\ y = -\frac{2}{5} \\ z = -1 \end{cases}$$

$$\begin{cases} 2x - 3y + z = 1 \\ x - y + 2z = 0 \\ 3x - 5y = 2 \end{cases}$$

$$D = \begin{vmatrix} 2 & -3 & 1 & 2 & -3 \\ 1 & -1 & 2 & 1 & -1 \\ 3 & -5 & 0 & 3 & -5 \end{vmatrix} = 0 - 18 - 5 - (-3 - 20 + 0) = -18 - 5 + 3 + 20 = 0$$

$$\begin{cases} \frac{4}{3} + \frac{10}{3}y - 3y + z = 1 \\ \frac{2}{3} + \frac{5}{3}y - y + 2z = 0 \\ x = \frac{2}{3} + \frac{5}{3}y \end{cases} \quad \begin{cases} 4 + 10y - 3y + 3z = 3 \\ 2 + 5y - 3y + 6z = 0 \\ \text{---} \end{cases}$$

$$\begin{cases} y + 3z = -1 \\ 2y + 6z = -2 \\ \text{---} \end{cases}$$

$$\left(\frac{a}{a'} = \frac{1}{2}\right) = \left(\frac{b}{b'} = \frac{1}{2}\right) = \left(\frac{c}{c'} = \frac{1}{2}\right)$$

S. INDET.

H

$$\begin{cases} z - 2y = 5 \\ x + 2y - z = 1 \\ x - 2y + z = 3 \end{cases}$$

$$D = \left| \begin{array}{ccc|cc} 0 & -2 & 1 & 0 & -2 \\ 1 & 2 & -1 & 1 & 2 \\ 1 & -2 & 1 & 1 & -2 \end{array} \right| = 0 + 2z - (z + 0z) = 0$$

$$\begin{cases} z = 5 + 2y \\ x + 2y - 5 - 2y = 1 \\ x - 2y + 5 + 2y = 3 \end{cases} \begin{cases} x = 6 \\ x = -2 \end{cases} \quad \text{S. IMPOSSIBILE}$$