

NOME Francesca Camilla

COGNOME Gentile

INDIRIZZO

TELEFONO

EMAIL

SCUOLA Liceo Scientifico IA

CLASSE

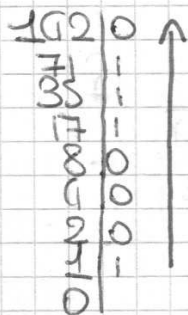
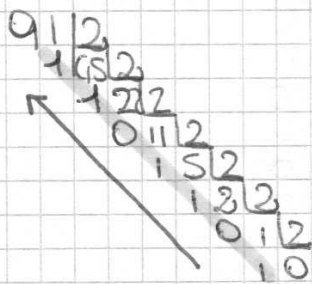
MATERIA MATEMATICA (M)

ANNO SCOLASTICO 2023/2024

SISTEMA BINARIO

Passaggio dal sistema decimale al binario

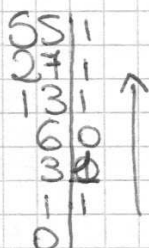
$$(91)_{10} = (1011011)_2$$



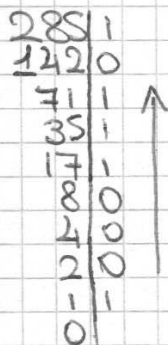
$$(91)_{10} = (10001110)_2$$

esercizi

$$(110111)_2 = (1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0)_{10} = (32 + 16 + 0 + 4 + 2 + 1)_{10} = (55)_{10}$$



$$(1000111011)_2 = (1 \cdot 2^8 + 0 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0)_{10} = (256 + 16 + 8 + 4 + 1)_{10} = (285)_{10}$$



OPERAZIONI NEL SISTEMA BINARIO

addizione

$$\begin{array}{r} 1011 + \\ 1111 = \\ \hline 1000101 \end{array}$$

$$\begin{array}{r} 1011 + \\ 1111 = \\ \hline 1000101 \end{array}$$

D	B
0	0
1	1
2	10
3	11
4	100
5	101

OPERAZIONI SISTEMA BINARIO

ADDIZIONE

$$\begin{array}{r}
 \text{11 10 10} \\
 1111 + \rightarrow 15 \\
 101 \quad 5 \\
 111 \quad 7 \\
 \hline
 1111 \quad 15 \\
 \hline
 101010 \quad 42
 \end{array}$$

1° METODO

D	R
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000

$$(101010)_2 = (1 \cdot 2^5 + 1 \cdot 2^3 + 1 \cdot 2^1)_{10} = (32 + 8 + 2)_{10} = (42)_{10}$$

$$\begin{array}{r}
 \text{11 11 10 1} \\
 111101 + \rightarrow 61 \\
 11110 \quad 30 \\
 11111 \quad 31 \\
 \hline
 101111 = 47 \\
 \hline
 10101001 = 169
 \end{array}$$

$$(111101)_2 = (1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1)_{10} = (32 + 16 + 8 + 4 + 1)_{10} = (61)_{10}$$

$$(11110)_2 = (1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1)_{10} = (16 + 8 + 4 + 2)_{10} = (30)_{10}$$

$$(101111)_2 = (1 \cdot 2^5 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2 + 1)_{10} = (32 + 8 + 4 + 2 + 1)_{10} = (47)_{10}$$

$$(10101001)_2 = (1 \cdot 2^7 + 1 \cdot 2^5 + 1 \cdot 2^3 + 1)_{10} = (128 + 32 + 8 + 1)_{10} = (169)_{10}$$

MOLTIPLICAZIONE

$$\begin{array}{r}
 1011 \cdot \\
 \hline
 101 \\
 \\
 1011 \\
 0000 - \\
 \hline
 1011 - - \\
 \hline
 110111
 \end{array}$$

$$\begin{array}{r}
 11111 \cdot \\
 \hline
 1110 \\
 \\
 101 \\
 1000000 \\
 1011111 - \\
 1011111 - - \\
 \hline
 11111 - - - \\
 \hline
 110110010
 \end{array}$$

SOTTRAZIONE:

$$\begin{array}{r} 11111- \\ \underline{1001} \\ 10110 \end{array}$$

Sottrazione "facile"

$$\begin{array}{r} \overset{1}{2} \overset{10}{10} \overset{1}{10} \\ 101001- \end{array}$$

$$\begin{array}{r} 1110 \\ \underline{11011} \end{array}$$

$$\begin{array}{r} \overset{0}{10} \overset{10}{10} \overset{10}{10} \overset{10}{10} \overset{10}{10} \\ 1000000- \\ \underline{11011} \\ 100101 \end{array}$$

$$\begin{array}{r} \overset{0}{10} \overset{10}{10} \overset{10}{10} \overset{10}{10} \\ 110000- \end{array}$$

$$\begin{array}{r} 101111 \\ \underline{0001} \end{array}$$

ADDIZIONE

$$\begin{array}{r} 1. \quad \begin{array}{r} 10110+ \\ 111001 \\ \hline 1001111 \end{array} \end{array}$$

$$\begin{array}{r} 2. \quad \begin{array}{r} 11110+ \\ 11100 \\ \hline 111010 \end{array} \end{array}$$

$$\begin{array}{r} 3. \quad \begin{array}{r} 10110+ \\ 110011 \\ \hline 1001001 \end{array} \end{array}$$

$$\begin{array}{r} 4. \quad \begin{array}{r} 11000+ \\ 1010+ \\ \hline 100111 \end{array} \end{array}$$

$$\begin{array}{r} 5. \quad \begin{array}{r} 1111+ \\ 1111+ \\ \hline 101 \end{array} \end{array}$$

$$\begin{array}{r} 6. \quad \begin{array}{r} 11111+ \\ 11111+ \\ \hline 1010101 \end{array} \end{array}$$

1011101

1010101

SOTTRAZIONE

$$\begin{array}{r} 1. \quad \begin{array}{r} 11110- \\ 110 \\ \hline 11000 \end{array} \end{array}$$

$$\begin{array}{r} 2. \quad \begin{array}{r} 10110- \\ 10 \\ \hline 10100 \end{array} \end{array}$$

$$\begin{array}{r} 3. \quad \begin{array}{r} 100110- \\ 1001 \\ \hline 11101 \end{array} \end{array}$$

$$\begin{array}{r} 4. \quad \begin{array}{r} 1000000- \\ 100110 \\ \hline 11010 \end{array} \end{array}$$

$$\begin{array}{r} 5. \quad \begin{array}{r} 11000000- \\ 1111 \\ \hline 1010001 \end{array} \end{array}$$

$$\begin{array}{r} 6. \quad \begin{array}{r} 1111000- \\ 111011 \\ \hline 111101 \end{array} \end{array}$$

MULTIPLICATIONE

$$\begin{array}{r}
 1. \quad 1101. \\
 \quad \underline{111} \\
 111 \\
 1101 \\
 1101- \\
 \underline{1101--} \\
 1011011
 \end{array}$$

$$\begin{array}{r}
 2. \quad 11110. \\
 \quad \underline{1000} \\
 00000 \\
 00000- \\
 00000-- \\
 \underline{11110---} \\
 11110000
 \end{array}$$

$$\begin{array}{r}
 3. \quad 1110. \\
 \quad \underline{1111} \\
 10101110 \\
 1110- \\
 1110-- \\
 \underline{1110---} \\
 11010010
 \end{array}$$

$$\begin{array}{r}
 4. \quad 10110. \\
 \quad \underline{1001} \\
 1110110 \\
 00000- \\
 00000-- \\
 \underline{10110---} \\
 11000110
 \end{array}$$

$$\begin{array}{r}
 5. \quad 11111. \\
 \quad \underline{111} \\
 10111111 \\
 11111- \\
 \underline{11111--} \\
 11011001
 \end{array}$$

$$\begin{array}{r}
 6. \quad 100101. \\
 \quad \underline{1011} \\
 11 \\
 100101 \\
 100101- \\
 000000-- \\
 \underline{100101---} \\
 110010111
 \end{array}$$

OPERAZIONI S. B.

$$\begin{array}{r}
 1010 \\
 \underline{110} \\
 0000 \\
 1010 - \\
 \underline{1010 -} \\
 111100
 \end{array}$$

$$\begin{array}{r}
 1111x \\
 \underline{110} \\
 10000 \\
 1111- \\
 \underline{1111-} \\
 1011010
 \end{array}$$

$$\begin{array}{r}
 1010101010 \\
 \underline{10000000-}
 \end{array}$$

RIPASSO

$$\begin{array}{r}
 11101 \\
 \underline{100011}
 \end{array}$$

DIVISIONE

$$\begin{array}{r}
 \overbrace{110110110} \\
 \underline{101} \\
 11 \\
 \underline{00} \\
 111 \\
 \underline{101} \\
 100 \\
 \underline{000} \\
 1001 \\
 \underline{101} \\
 1001 \\
 \underline{101} \\
 1000 \\
 \underline{101} \\
 11
 \end{array}
 \quad
 \begin{array}{r}
 101 \\
 \underline{1010111}
 \end{array}$$

$$\begin{array}{r}
 \overline{1001} \overline{110} \overline{110} \\
 \underline{110} \quad | \quad 110 \\
 111 \\
 \underline{110} \\
 11 \\
 \underline{00} \\
 110 \\
 \underline{110} \\
 0
 \end{array}$$

$$\begin{array}{r}
 0 \ 10 \ 10 \\
 \underline{1001} \\
 110 \\
 11
 \end{array}$$

$$\begin{array}{r}
 \overline{1001} \overline{101} \overline{101} \\
 \underline{101} \quad | \quad 111 \\
 1001 \\
 \underline{101} \\
 1000 \\
 \underline{101} \\
 11
 \end{array}$$

$$\begin{array}{r}
 0 \ 10 \\
 \underline{1001} - \\
 101 \\
 \underline{100} \\
 0 \ 10 \ 10 \ 10 \\
 \underline{1000} - \\
 101 \\
 11
 \end{array}$$

5 addizioni

$$\begin{array}{r}
 100110 + 38 \\
 111 + 7 \\
 \hline
 10111 = 23 \\
 1000100 = 68
 \end{array}$$

0
1
10
11
100
101
110
111
1000

verifica

$$(100110)_2 = (1 \cdot 2^5 + 1 \cdot 2^2 + 1 \cdot 2)_2 = (32 + 4 + 2)_{10} = (38)_{10}$$

$$(10111)_2 = (1 \cdot 2^4 + 1 \cdot 2^2 + 1 \cdot 2 + 1)_{10} = (16 + 4 + 2 + 1)_{10} = (23)_{10}$$

$$(1000100)_2 = (1 \cdot 2^6 + 1 \cdot 2^2)_{10} = (64 + 4)_{10} = (68)_{10}$$

$$\begin{array}{r}
 10111 + 23 \\
 10010 + 18 \\
 \hline
 1111 = 15 \\
 111000 = 56
 \end{array}$$

verifica

$$(10111)_2 = (1 \cdot 2^4 + 1 \cdot 2^2 + 1 \cdot 2 + 1)_{10} = (16 + 4 + 2 + 1)_{10} = (23)_{10}$$

$$(10010)_2 = (1 \cdot 2^4 + 1 \cdot 2)_{10} = (16 + 2)_{10} = (18)_{10}$$

$$(111000)_2 = (1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3)_{10} = (32 + 16 + 8)_{10} = (56)_{10}$$

$$(1111)_2 = (1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2 + 1)_{10} = (8 + 4 + 2 + 1)_{10} = (15)_{10}$$

$$\begin{array}{r}
 10111 + 23 + \\
 101 + 5 \\
 \hline
 11100 = 28
 \end{array}$$

verifica

$$(10111)_2 = (1 \cdot 2^4 + 1 \cdot 2^2 + 1 \cdot 2 + 1)_{10} = (16 + 4 + 2 + 1)_{10} = (23)_{10}$$

$$(11100)_2 = (1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2)_{10} = (16 + 8 + 4)_{10} = (28)_{10}$$

$$\begin{array}{r}
 4. \quad 10000 + \quad 16 \\
 \quad 1000 \quad 8 \\
 \hline
 \quad 10001 \quad 17 \\
 101001 \quad 24
 \end{array}$$

verifica

$$(10000)_2 = (2^4)_{10} = (16)_{10}$$

$$(1000)_2 = (2^3)_{10} = (8)_{10}$$

$$(10001)_2 = (2^4 + 1)_{10} = (16 + 1)_{10} = (17)_{10}$$

$$(101001)_2 = (2^5 + 2^3 + 1)_{10} = (32 + 8 + 1)_{10} = (41)_{10}$$

$$\begin{array}{r}
 5. \quad 10001 + \quad 17 + \\
 \quad 1011 \quad 11 \\
 \hline
 \quad 11100 \quad 28
 \end{array}$$

$$(1011)_2 = (1 \cdot 2^3 + 1 \cdot 2 + 1)_{10} = (8 + 2 + 1)_{10} = (11)_{10}$$

5 moltiplicazioni

$$\begin{array}{r}
 1. \quad 100001 \cdot \quad 33 \cdot \\
 \quad 1011 \quad 11 \\
 \hline
 \quad 100001 \\
 \quad 100001 - \\
 \quad 000000 - - \\
 \hline
 100001 - - - \\
 101101011
 \end{array}$$

$$(100001)_2 = 32 + 1 = 33$$

$$(1011)_2 = 8 + 2 + 1 = 11$$

$$(101101011)_2 = 256 + 64 + 32 + 8 + 2 + 1 = 363$$

$$\begin{array}{r}
 2. \quad 1011 \cdot \quad 11 \cdot \\
 \quad 11 \quad 3 \\
 \hline
 \quad 1011 \quad 33 \\
 \quad 1011 - \\
 \hline
 100001
 \end{array}$$

$$(1011)_2 = 8 + 2 + 1 = 11$$

$$(100001)_2 = 32 + 1 = 33$$

$$\begin{array}{r}
 3. \quad 101111. \\
 \quad \quad \underline{111} \\
 10101111 \\
 \quad \quad \underline{101111} \\
 101111- \\
 \quad \quad \underline{101111} \\
 101001001
 \end{array}
 \qquad
 \begin{array}{r}
 47. \\
 \quad \underline{7} \\
 329
 \end{array}$$

$$(101111)_2 = 32 + 8 + 4 + 2 + 1 = (47)_{10}$$

$$(101001001)_2 = (256 + 64 + 8 + 1)_{10} = (329)_{10}$$

$$\begin{array}{r}
 4. \quad 111. \\
 \quad \quad \underline{111} \\
 101111 \\
 \quad \quad \underline{111} \\
 111- \\
 \quad \quad \underline{111} \\
 110001
 \end{array}
 \qquad
 \begin{array}{r}
 7. \\
 \quad \underline{7} \\
 49
 \end{array}$$

$$(110001)_2 = (32 + 16 + 1) = (49)_{10}$$

$$\begin{array}{r}
 5. \quad 1010. \\
 \quad \quad \underline{11} \\
 1010 \\
 \quad \quad \underline{1010} \\
 11110
 \end{array}
 \qquad
 \begin{array}{r}
 10. \\
 \quad \underline{3} \\
 30
 \end{array}$$

$$(1010)_2 = (8 + 2)_2 = (10)_2$$

$$(11110)_2 = (16 + 8 + 4 + 2)_{10} = (30)_{10}$$

5 sottrazioni

$$\begin{array}{r}
 1. \quad 11111- \\
 \quad \quad \underline{1001} \\
 10110
 \end{array}
 \qquad
 \begin{array}{r}
 31- \\
 \quad \underline{9} \\
 22
 \end{array}$$

$$(11111)_2 = (16 + 8 + 4 + 2 + 1)_{10} = (31)_{10}$$

$$(10110)_2 = (16 + 4 + 2 + 1)_{10} = (23)_{10}$$

$$3. \quad \begin{array}{r} 2^0 2^1 2^2 2^3 2^4 \\ 10111- \\ \underline{1100} \\ 1011 \end{array} \quad \begin{array}{r} 23 \\ \underline{12} \\ 11 \end{array}$$

$$(10111)_2 = (16+4+2+1)_{10} = (23)_{10}$$

$$(1100)_2 = (8+4)_{10} = (12)_{10}$$

$$(1011)_2 = (8+2+1)_{10} = (11)_{10}$$

$$3. \quad \begin{array}{r} 2^0 2^1 2^2 2^3 2^4 2^5 2^6 2^7 \\ 1000000- \\ \underline{1001} \\ 110111 \end{array} \quad \begin{array}{r} 69 \\ \underline{9} \\ 55 \end{array}$$

$$(1000000)_2 = (2^6)_{10} = 64$$

$$(110111)_2 = (32+16+4+2+1)_{10} = 55$$

$$4. \quad \begin{array}{r} 2^0 2^1 2^2 2^3 \\ 1000- \\ \underline{111} \\ =001 \end{array} \quad \begin{array}{r} 8- \\ \underline{7} \\ 1 \end{array}$$

$$5. \quad \begin{array}{r} 2^0 2^1 2^2 2^3 2^4 2^5 2^6 2^7 \\ 1000001- \\ \underline{101111} \\ 0010010 \end{array} \quad \begin{array}{r} 65- \\ \underline{47} \\ 18 \end{array}$$

$$(101111)_2 = (32+8+4+2+1)_{10} = 47$$

$$(10010)_2 = (16+2)_{10} = 18$$

5 divisioni

$$1. \quad \begin{array}{r} \overbrace{11000}^{1000} \overbrace{11}^{10} \\ \underline{11} \\ 00 \\ \underline{00} \\ 00 \\ \underline{00} \\ 00 \\ \underline{00} \\ 00 \\ \underline{00} \\ 00 \end{array}$$

$$(11000)_2 = (16+8)_{10} = 24 : 3 = 8$$

$$3. \quad \begin{array}{r} \overbrace{11110}^{110} \overbrace{110}^{101} \\ \underline{110} \\ 11 \\ \underline{11} \\ 00 \\ \underline{00} \\ 00 \\ \underline{00} \\ 00 \end{array}$$

$$(11110)_2 = (16+8+4+2)_{10} = (30)_{10}$$

$$30 : 6 = 5$$

$$\begin{array}{r|l}
 \overline{110011} & \overline{1001} \\
 \underline{1001} & 101 \\
 111 & \\
 \underline{000} & \\
 1111 & \\
 \underline{1001} & \\
 110 &
 \end{array}$$

$$\begin{array}{r}
 \\
 1100- \\
 \underline{1001} \\
 11
 \end{array}$$

$$(110011)_2 = (32 + 16 + 2 + 1)_{10} = (51)_{10}$$

$$(1001)_2 = (9)_{10}$$

$$(101)_2 = (5)_{10}$$

$$(110)_2 = (6)_{10}$$

$$\begin{array}{r|l}
 \overline{51} & \overline{9} \\
 \underline{45} & 5 \\
 6 &
 \end{array}$$

$$\begin{array}{r|l}
 \overline{101100} & \overline{100} \\
 \underline{100} & 1011 \\
 11 & \\
 \underline{0} & \\
 110 & \\
 \underline{100} & \\
 100 & \\
 \underline{100} & \\
 0 &
 \end{array}$$

$$\begin{array}{r|l}
 \overline{26} & \overline{6} \\
 \underline{26} & 11 \\
 0 &
 \end{array}$$

$$(101100)_2 = (32 + 8 + 6)_{10} = (46)_{10}$$

$$(100)_2 = (4)_{10}$$

$$(1011)_2 = (8 + 2 + 1)_{10} = (11)_{10}$$

$$\begin{array}{r|l}
 \overline{100111} & \overline{111} \\
 \underline{111} & 101 \\
 101 & \\
 \underline{000} & \\
 1011 & \\
 \underline{111} & \\
 100 &
 \end{array}$$

$$\begin{array}{r|l}
 \overline{39} & \overline{7} \\
 \underline{35} & 5 \\
 4 &
 \end{array}$$

$$\begin{array}{r}
 \\
 1001- \\
 \underline{111} \\
 10
 \end{array}$$

$$\begin{array}{r}
 \\
 1010- \\
 \underline{111} \\
 0011
 \end{array}$$

$$(100111)_2 = (32 + 4 + 2 + 1)_{10} = (39)_{10}$$

SISTEMA ESADECIMALE

Il sistema esadecimale usa 16 simboli

(D) 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

(E) 0 1 2 3 4 5 6 7 8 9 A B C D E F 10 11 12

(D) 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37

(E) 13 14 15 16 17 18 19 1A 1B 1C 1D 1E 1F 20 21 22 23 24 25

(D) 38 39 40 41 42 43 44 45 46 47 48 49 50 51

(E) 26 27 28 29 2A 2B 2C 2D 2E 2F 30 31 32 33

D

(E) 9F A0 A1 A2 A3 ... AA AB AC AD AE AF B0 B1

trasformazione

$$(12A)_{16} = (1 \cdot 16^2 + 2 \cdot 16 + A \cdot 16^0)_{10} = (256 + 32 + 10)_{10} = (298)_{10}$$

$$(2AE)_{16} = (2 \cdot 16^2 + A \cdot 16 + E \cdot 16^0)_{10} = (512 + 160 + 14)_{10} = (686)_{10}$$

$$(1BOC)_{16} = (1 \cdot 16^3 + B \cdot 16^2 + 0 \cdot 16^1 + C \cdot 16^0)_{10} = (4096 + 2816 + 12)_{10} = (6924)_{10}$$

$$\begin{array}{r|l} 298 & 10 \\ 18 & 2 \\ \hline & 0 \end{array} \uparrow (12A)_{16}$$

$$\begin{array}{r|l} 686 & 14 \\ 42 & 10 \\ \hline & 2 \\ & 0 \end{array} \uparrow (2AE)_{16}$$

$$\begin{array}{r|l} 6924 & 12 \\ 132 & 0 \\ \hline & 27 \\ & 11 \\ & 1 \\ & 0 \end{array} \uparrow (1BOC)_{16}$$

CALCOLO

$$\begin{array}{r|l} 432 & 16 \\ 32 & 27 \\ \hline 112 & \\ 112 & \\ \hline = & \end{array}$$

$$\begin{array}{r|l} 27 & 16 \\ 16 & 11 \\ \hline & \end{array}$$

$$\begin{array}{r|l} 1 & 16 \\ 0 & 0 \\ \hline 1 & \end{array}$$

$$\begin{array}{r|l} 298 & 16 \\ 16 & 18 \\ \hline 138 & \\ 128 & \\ \hline 10 & \end{array}$$

$$\begin{array}{r|l} 18 & 16 \\ 16 & 1 \\ \hline 2 & \end{array}$$

$$\begin{array}{r|l} 1 & 16 \\ 0 & 0 \\ \hline 1 & \end{array}$$

$$\begin{array}{r|l} 686 & 16 \\ 64 & 42 \\ \hline 46 & \\ 32 & \\ \hline 14 & \end{array}$$

$$\begin{array}{r|l} 42 & 16 \\ 32 & 27 \\ \hline 10 & \end{array}$$

$$\begin{array}{r|l} 2 & 16 \\ 0 & 0 \\ \hline 2 & \end{array}$$

$$\begin{array}{r|l} 6924 & 16 \\ 64 & 432 \\ \hline 4096 & \\ 282 & \\ \hline 44 & \\ 32 & \\ \hline 12 & \end{array}$$

$$S_4 = \{0, 1, 2, 3\}$$

$$\begin{aligned} (1203)_4 &= (1 \cdot 4^3 + 2 \cdot 4^2 + 0 \cdot 4 + 3 \cdot 1)_{10} = \\ &= (64 + 2 \cdot 16 + 3)_{10} = \\ &= (64 + 32 + 3)_{10} = (99)_{10} \end{aligned}$$

$$\begin{array}{r|l} 99 & 3 \\ 20 & 0 \\ \hline 6 & 2 \\ - & \\ 0 & \end{array} \quad \uparrow \quad (1203)_4$$

10	0
11	1
12	2
13	3
14	0
15	1
16	2
17	3
18	0
19	1
20	2
21	3
22	0
23	1
24	2
25	3
26	0
27	1
28	2
29	3
30	0
31	1
32	2
33	3
34	0
35	1
36	2
37	3
38	0
39	1
40	2
41	3
42	0
43	1
44	2
45	3
46	0
47	1
48	2
49	3

CALCOLI

$$\begin{array}{r} 10 \\ \hline 20 \\ \hline 30 \\ \hline 40 \\ \hline 50 \\ \hline 60 \\ \hline 70 \\ \hline 80 \\ \hline 90 \end{array}$$